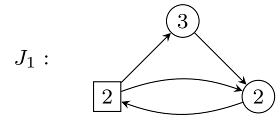
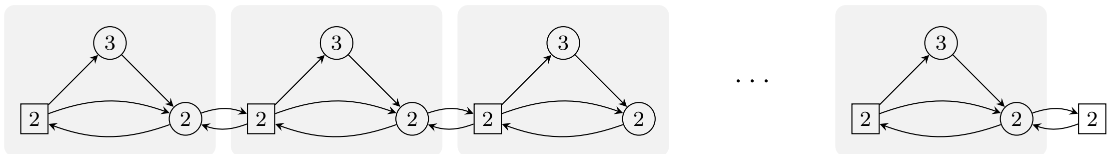
Contributor: K.S. Thejaswini, M.Sc CS

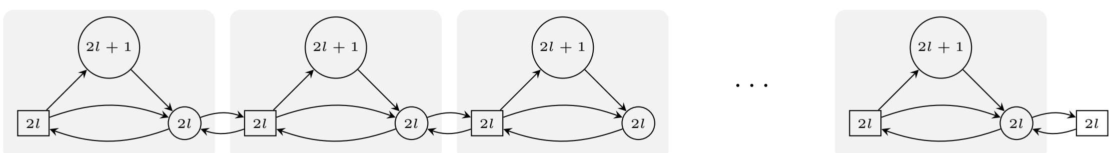
1. Compute the least progress progress measure for the graph J_1 shown in the figure below.



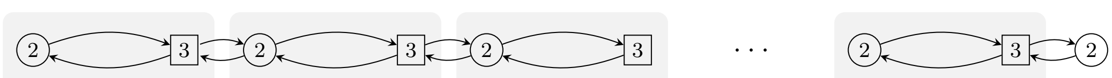
2. Compute the least progress measure for the graph $G_{1,b}$ which is formed by connecting b "blocks" of the above graph J_1 and an extra vertex, in the manner illustrated in the figure.



3. Let $G_{l,b}$ be the graph constructed as $G_{1,b}$ above, with priority 2 replaced by $2l$ and priority 3 replaced by $2l + 1$. Compute the least progress measure for $G_{l,b}$.



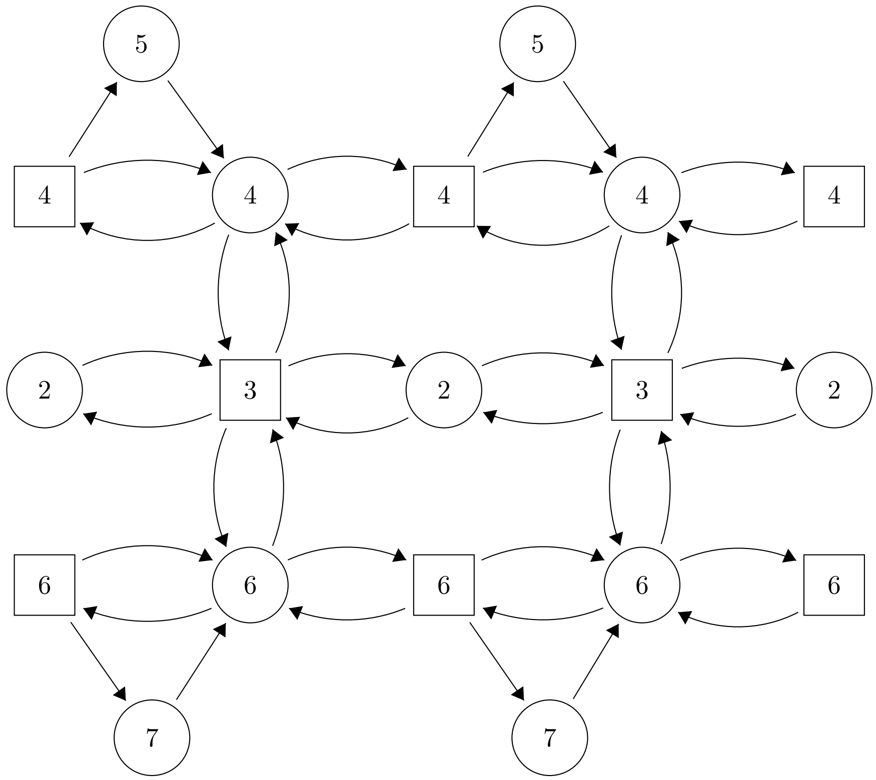
4. Compute least progress measure for the graph $H_{1,b}$ shown below:



5. We now construct a graph $P_{k,b}$. The graph $P_{k,b}$ consists of the graphs $G_{l,b}$ for $l \in \{2, \dots, k\}$ and $H_{1,b}$ connected by additional edges which we describe now.

   Each block in $G_{l,b}$ contains a Player 0's vertex with priority $2l$: let $u_{l,i}$ be this vertex in block i of $G_{l,b}$. Similarly, each block in $H_{1,b}$ contains a Player 1 vertex with priority 3: let v_i be this vertex in block i of $H_{1,b}$. Add edges $u_{l,i} \to v_i$ and $v_i \to u_{l,i}$ for every $i \in \{1, \dots, b\}$ and every $l \in \{2, \dots, k\}$.

   The graph $P_{k,b}$ for $k = 3, b = 2$ is shown below:



For every graph $P_{k,b}$ obtained as above, show that the algorithm *progress-measure-lifting($P_{k,b}$)* would make at least $(b+1)^k$ lifts before termination.

6. Using the examples above, conclude that there are instances where the *progress-measure-lifting* algorithm, on graphs of size n would take $O(\lceil n/d \rceil^{d/2})$, making the naive computation of the run time bound a tight one.

7. Let $\mathcal{T}_n$ be the (directed) binary tree of depth n. Assume that the leaves of $\mathcal{T}_n$ have self loops. All non-leaf nodes have priority 1. Among the leaf nodes, some (arbitrary number) of them have priority 1 and the others have priority 2.

   i) Assuming that all nodes are Player 0's nodes, what is the progress measure computed by *progress-measure-lifting($\mathcal{T}_n$)*.

   ii) Assuming that all nodes are Player 1's nodes, what is the progress measure computed by *progress-measure-lifting*($\mathcal{T}_n$).

   How does your answer depend on the arrangement of priorities 1 and 2 among leaf nodes?

8. Let G be a graph on the vertex set $\{v_1, v_2, \ldots, v_{2n-1}, v_{2n}\}$. Let the partition of vertices V be such that $V_0 = \{v_i \mid i \text{ is odd } \}$ and $V_1 = \{v_j \mid j \text{ is even } \}$. Let the parity of vertex v_i be i. Now the edges in the graph are defined as follows: There is an edge from v_i to v_j iff $i \bmod 2 \neq j \bmod 2$. Compute the progress measure for these graphs and hence the winning regions and strategies for player 0 and 1.