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1. Show that in a rooted tree with internal nodes (non-leaf nodes) having at least two children, the number of leaves is greater than the number of internal nodes .
2. Prove or disprove the following.
  - i) The union of two dominions of a player is also a dominion of that player .
  - ii) The intersection of two dominions of a players is also a dominion of that player .
3. Design a family of games  $\mathcal{G}_n$  with  $n$  vertices such that there are no dominions of size at most  $\sqrt{2n}$  in  $\mathcal{G}_n$ .