

PROBLEM SET 2

DISCRETE MATHEMATICS 2020-21

CHENNAI MATHEMATICAL INSTITUTE

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“ May the force be with you.

”

Obi-Wan Kenobi, *Jedi Master*

Problem 4. A long time ago, in a galaxy far, far away, a shelf holds 12 books in a row. How many ways are there to choose five books so that no two adjacent books are chosen?

Problem 5. Find the probability that when a fair coin is flipped five times tails comes up exactly three times, the first and last flips come up tails, or the second and fourth flips come up heads.

Problem 6. Suppose that a large family has 14 children, including two sets of identical triplets, three sets of identical twins, and two individual children. How many ways are there to seat these children in a row of chairs if the identical triplets or twins cannot be distinguished from one another?

Problem 1. How many solutions are there to the equation $x_1 + x_2 + x_3 + x_4 + x_5 = 21$, where $x_i, i = 1, 2, 3, 4, 5$ is a non-negative integer such that:

- (a) $x_1 \geq 1$?
- (b) $x_i \geq 2$ for $i = 1, 2, 3, 4, 5$?
- (c) $0 \leq x_1 \leq 10$?
- (d) $0 \leq x_1 \leq 3, 1 \leq x_2 < 4$, and $x_3 \geq 15$?

Problem 2. How many ways are there to distribute 12 indistinguishable balls into six distinguishable bins?

Problem 3. Give a combinatorial argument for the following identity:

$$\sum_{k=0}^n \binom{n}{k}^2 = \binom{2n}{n}$$

“ Never tell me the odds.

”

Han Solo, *Rebel Pilot*

Problem 7. In a survey of 270 college students, it is found that 64 like brussels sprouts, 94 like broccoli, 58 like cauliflower, 26 like both brussels sprouts and broccoli, 28 like both brussels sprouts and cauliflower, 22 like both broccoli and cauliflower, and 14 like all three vegetables. How many of the 270 students do not like any of these vegetables?

Problem 8. A poker hand means a set of five cards selected at random from usual deck of playing cards.

(a) Find the probability that it is a Royal Flush - means that it consists of ten, jack, queen, king, ace of the same suit. (b) Find the probability that it is four of a kind - means that there are four cards of equal face value. (c) Find the probability that it is a full house - means that it consists of one pair and one triple of cards with equal face values. (d) Find the probability that it is a straight - means that it consists of five cards in a sequence regardless of suit. (e) Find the probability that it consists of three cards of equal face value and two other cards but not a full house. (f) Find the probability that it consists of two distinct pairs and another card but does not fall into previous categories. (g) Find the probability that it consists of a pair and three other cards but does not fall into previous categories.

(Feel free to check out Poker hands for clarification in case of ambiguity.)

Problem 9. A student has three mangos, two papayas, and two kiwi fruits. If the student eats one piece of fruit each day, and only the type of fruit matters, in how many different ways can these fruits be consumed?

Solo. Suppose a person wants to send four different e-mails to the aliases `hostel@cmi.ac.in`, `cmiall@cmi.ac.in`, `ug@cmi.ac.in` and `msc@cmi.ac.in`. Due to negligence, he shuffles the recipient addresses for his mails. In how many ways can he mess up so that no mail gets sent to the intended recipient?

“ I’ve got a bad feeling about this.

”

Basically everyone

Rogue One. In how many ways can eight rooks be placed on a chess board so that no rooks attack each other and no rooks are placed on the white diagonal?