DISCRETE MATHEMATICS

LECTURE 6

Plan:

Propositional logic (contd.)

- Some problems

Example 1: Four siblings go shopping with their father.

- If Abhay gets shoes, then Asha does not get a necklace.
- If Arun gets a T-shirt, then Aditi gets bangles.
- If Abhay does not get shoes or Aditi gets bangles, the mother will be happy.
- Mother is not happy.

Prove that Asha did not get a necklace and Arun did not get a T-shirt.

PAUSE

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- If Abhay does not get shoes or Aditi gets bangles, the mother will be happy.
- Mother is not happy.

Prove that Asha did not get a necklace and Arun did not get a T-shirt.

p: Abhay gets shoes

q: Asha gets necklace

r: Arun gets T-shirt

s: Aditi gets bangles

m: mother is happy.

Premises:

1. $p \rightarrow \neg q$
2. $r \rightarrow s$
3. $\neg p \vee s \rightarrow m$
4. $\neg m$

To prove: $\neg q \wedge \neg r$

Proof: From 3 and 4, we can infer $\neg(\neg p \vee s)$

$p \wedge \neg s$

From 1, and 'p' we get $\neg q$

From 2 and $\neg s$, we get $\neg r$

Premises:

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To prove:

$\neg q \wedge \neg r$

Proof: From 3 and 4, we can infer $\neg(\underbrace{\neg p}_{a} \vee \underbrace{s}_{b})$

$p \wedge \neg s$

From 1, and 'p' we get $\neg q$

From 2 and $\neg s$, we get $\neg r$

Example 2: Five friends A, B, C, D, E have access to a chat room. Is it possible to determine who is chatting if the following information is known?

- Either A or B, or both are chatting.
- Either C or D, but not both, are chatting.
- If E is chatting, then C is also chatting.
- D and A are either both chatting or neither is.
- If B is chatting, then so are A and E.

Explain your reasoning.

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- If B is chatting, then so are A and E.

Explain your reasoning.

A: A is chatting

⋮

E: E is chatting.

1. $A \vee B$.
2. $C \vee D$.

2.1 $\neg(C \wedge D)$

3. $E \rightarrow C$

4. $D \rightarrow A$

4.1 $A \rightarrow D$

5. $B \rightarrow A \wedge E$

$(D \wedge A) \vee (\neg D \wedge \neg A)$

$D \leftrightarrow A$

1. $A \vee B$.
2. $C \vee D$.
2.1 $\neg(C \wedge D)$
3. $E \rightarrow C$
4. $D \rightarrow A$
4.1 $A \rightarrow D$
5. $B \rightarrow A \wedge E$

Assume $A = 1$

$\Rightarrow$ $D = 1$ (4.1)

$\Rightarrow$ $C = 0$ (2.1)

$\Rightarrow$ $E = 0$ (3)

$\Rightarrow$ $B = 0$ (5)

✓

Assume $A = 0$

$\Rightarrow$ $D = 0$ (4)

$\Rightarrow$ $C = 1$ (2)

$\Rightarrow$ $B = 0$ (5)

$\Rightarrow$ $B = 1$ (due to 1 and $A = 0$)

- a contradiction

- My assumption $A = 0$ is not possible.

Final answer: A and D are chatting.

1. $A \vee B$.

2. $C \vee D$.

2.1 $\neg(C \wedge D)$

3. $E \rightarrow C$

4. $D \rightarrow A$

4.1 $A \rightarrow D$

5. $B \rightarrow A \wedge E$

A	B	C	D	E	1	2	2.1	....	5
0	0	0	0	0					
0	0	0	0	1					
.	.	.	.	.					
1	0	0	1	0	1.	1.	1.	1.	1 1.
.	.	.	.	.					
|	1	|	1	1					

Example 3: Smullyan puzzle.

There is an island that has two kinds of inhabitants

- knights who always tell the truth
- knaves who always lie.

You encounter two people A and B. What are A and B if:

A says "B is a knight" and

B says "The two of us are opposite types".

Example 3: Smullyan puzzle.

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- knights who always tell the truth
- knaves who always lie.

You encounter two people A and B. What are A and B if
A says "B is a knight" and
B says "The two of us are opposite types".

A : A is knight

B : B is knight

1. $A \rightarrow B$
2. $\neg A \rightarrow \neg B$

$B \rightarrow [(A \wedge \neg B) \vee (\neg A \wedge B)]$

$\neg B \rightarrow [(A \wedge B) \vee (\neg A \wedge \neg B)]$

3. $B \rightarrow \underline{\neg A}$
4. $\neg B \rightarrow \neg A$

$A = 1$

$\rightarrow B = 1$ (1)

$\Rightarrow A = 0$ (3)

$\rightarrow$ contradiction.

$A = 0$

$\Rightarrow B = 0$ (2)

$\hookrightarrow$ consistent with 1, 3, 4

Hence

$A = 0 \wedge B = 0$

Both are knaves.

Example 4: There is an island that has two kinds of inhabitants

- knights who always tell the truth
- knaves who always lie.

You encounter two people A and B. What are A and B if:

4.1. A says "At least one of us is a knave" and
B says nothing.

4.2. A says "The two of us are both knights" and
B says "A is a knave"

4.3. A says "I am a knave or B is a knight" and
B says nothing.

4.4. Both A and B say "I am a knight".

4.5. A says "We are both knaves" and B says nothing.

4.1. A says "At least one of us is a knave" and
B says nothing.

$A \rightarrow \neg B$

$\neg A \rightarrow A \wedge B$ (contradiction)

A knight, B knave

4.2. A says "The two of us are both knights" and
B says "A is a knave"

1. $A \rightarrow A \wedge B$
2. $\neg A \rightarrow (B \vee \neg B)$
2. $B \rightarrow \neg A$
4. $\neg B \rightarrow A$

$A = 1 \Rightarrow B = 1$ (1)

$= A = 0$ (3)

contradiction

Assume $A = 0$

Assume $B = 0$ | Assume $B = 1$

$\Rightarrow A = 1$ (4)

contradiction

A knave, B knight.

4.3. A says "I am a knave or B is a knight" and
B says nothing

$A \rightarrow \neg A \vee B$

$A \rightarrow B$

$\neg A \rightarrow A \wedge \neg B$ (contradiction)

A

$A = 1, \quad B = 1$

Both A, B are knights.

4. 4. Both A and B say "I am a knight".

A → A	B → B
¬A → ¬A	¬B → ¬B

All are possible

4.5. A says "We are both knaves" and B says nothing.

$A \rightarrow (\neg A \wedge \neg B)$ (contradiction)

$\neg A \rightarrow B$

A knave, B knight