

#### LECTURE 5

#### Plan:

#### Reference:

- Introduction to logic
- Rules for natural deduction

Sections 1.1 and 1.2.1 of

LOGIC IN COMPUTER SCIENCE

(second edition)

by Michael HUTH & Mark RYAN Goal: a mechanism to infer "statements" from a given

set of "statements"

Example 1: IF the train arrives late AND

there are NO taxis at the station,

THEN John is late for his meeting.

John is NOT late for his meeting.

The train did arrive late.

What can you infer?

There are taxis at the station

Example 2: IF it is raining AND

Jane does NOT have her umbrella with her

THEN she will get wet.

Jane is NOT wet.

It is raining.

What can you infer?

Jane has her umbrella with her.

#### Both examples have the same "smeture".

IFthe train arrive lateANDIFit is tainingANDthere are no taxis at the station,Jane doe not have her umbrelle with herTHENJohn is late for his meeting.THENshe will get wetJohn is NOT late for his meeting.Jane is NOT wet.The train did arrive late.If is raining.

### IF p AND NOT Q THEN Y, NOT R. p.

## THEREFORE 9.

#### Both examples have the same "smeture".

 IF
 the train arrive late
 AND
 IF
 it is taining
 AND

 there are no taxis at the station,
 Jane does not have her umbrelle with her

 THEN
 John is late for his meeting.
 THEN
 She will get wet.

 John is NOT late for his meeting.
 Jane is NOT wet.

 The train did arrive late.
 It is raining.

IF p AND NOT Q THEN Y, NOT X. p.

#### THEREFORE 9.

PROPOSITIONS: the train arrived late, } p Propositional it is raining variables there are taxis at the station } g Jane that her umbrella with her

#### PROPOSITIONAL LOGIC:

a system of reasoning over propositional statements.

Basic blocks: Propositional Variable.

From propositions we can form compound statements

using operators.

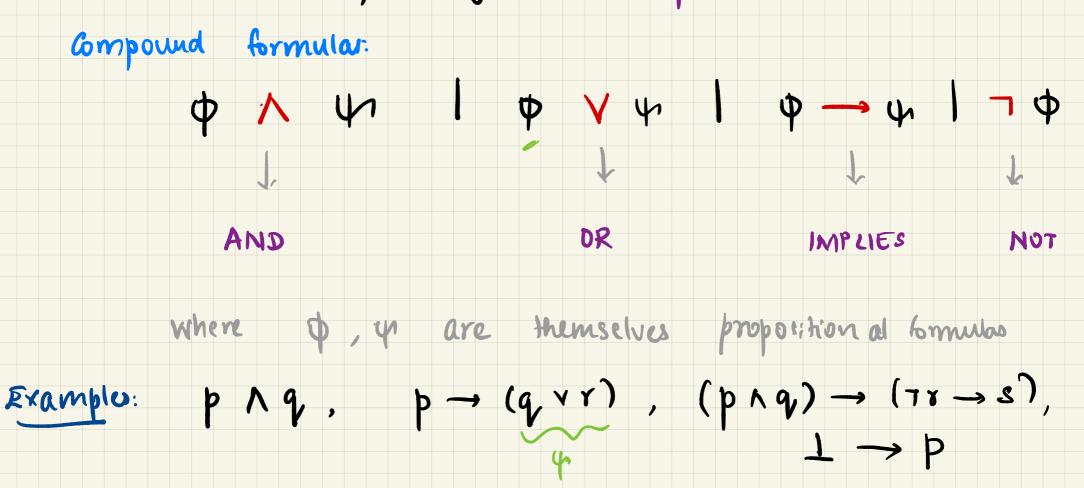
IF P AND NOT Q THEN Y

#### Propositions: P, q, r, s, t, etc

Propositional formulas:

Basic formular all propositions

+ 2 special symbols T (top) and 1 (bottom)

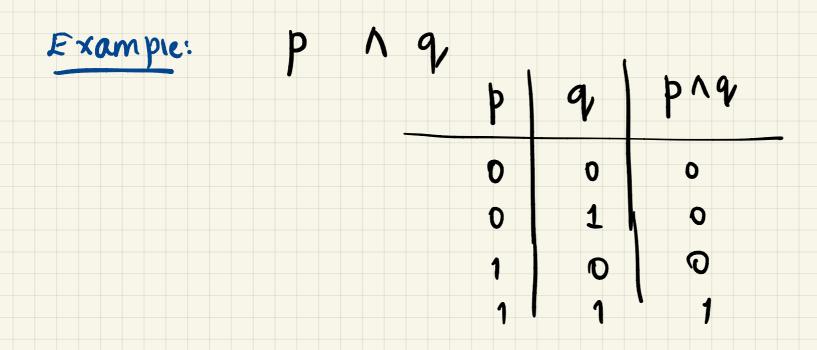


#### Meaning of propositional formulas:

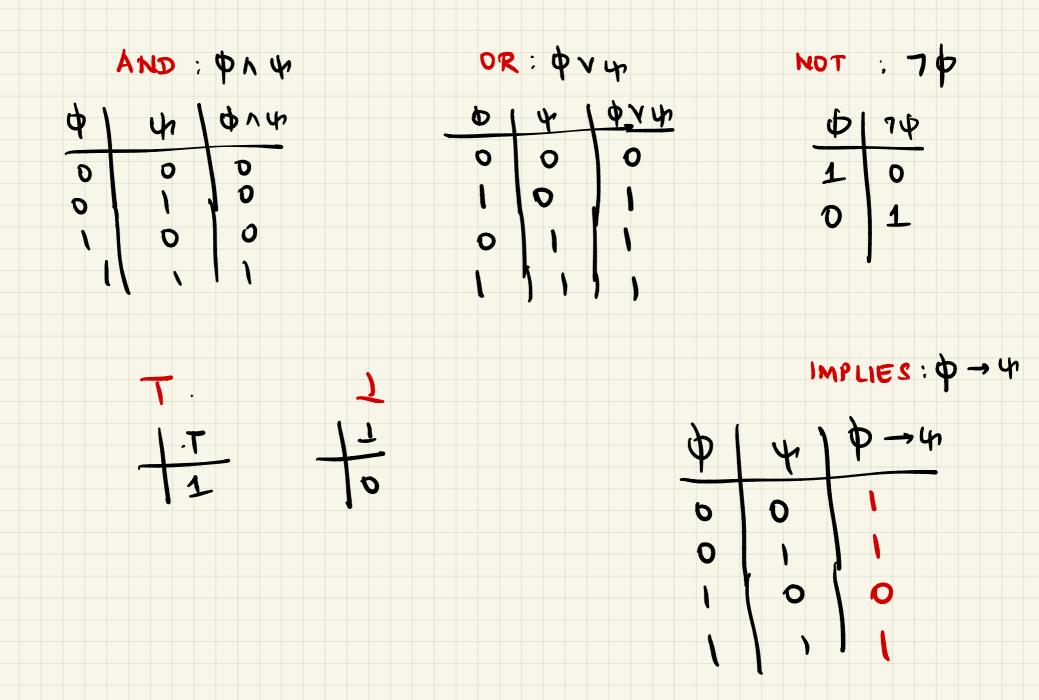
Propositional variables can take either true or false

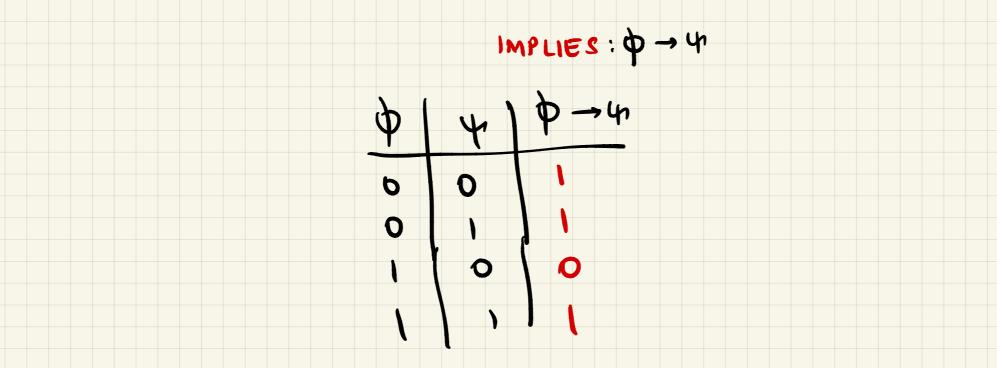
- also denoted as 1 or 0

Meaning of propositional formulas is given by a truth table



Trum tables inclued by the operators:



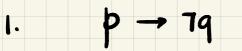


#### IF & is true THEN 41 is true

φ → 4 is equivelent to 7φ × 4

Truth table of more complex formulas:

Construct the truth tables for the following formulas.



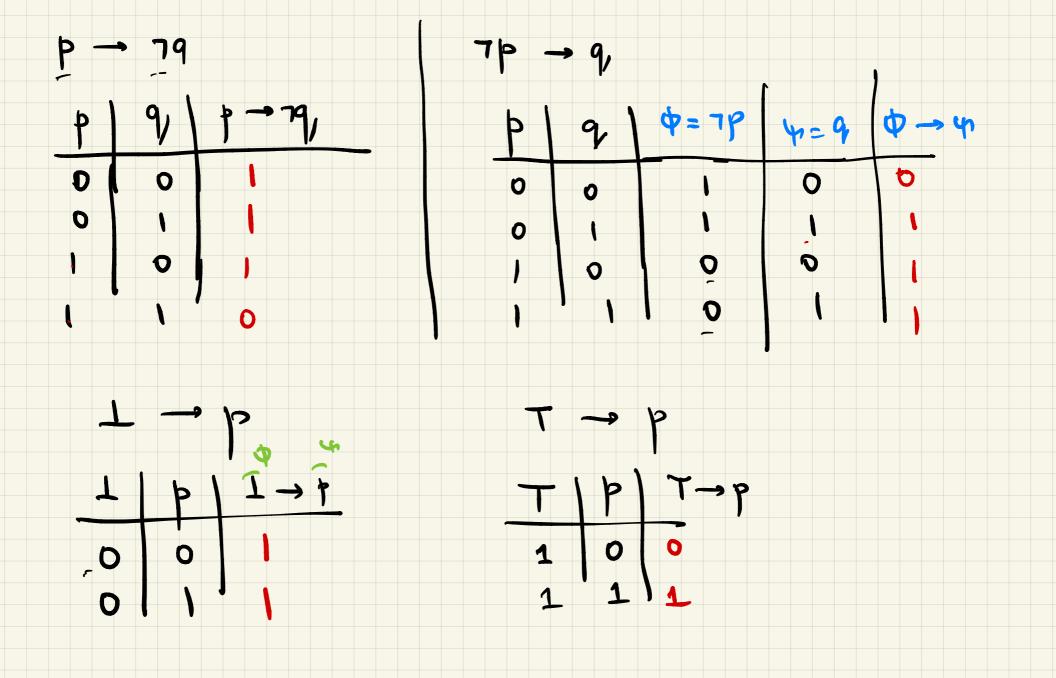
2.  $Tp \rightarrow q$ 

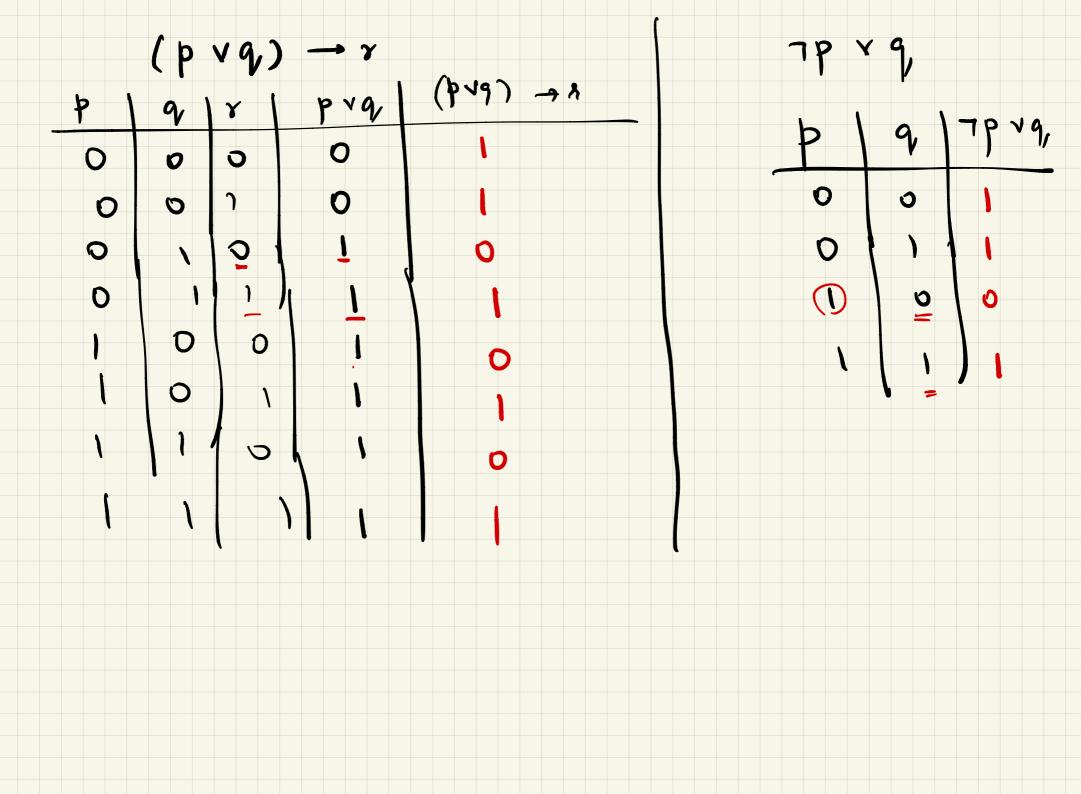


 $4. \quad \bot \rightarrow p$ 



6. TP V 9/







## $p \land q \rightarrow Tr \lor S - How to read it when there are no brackets?$

# $\rightarrow$ stands for: $(p \land q) \rightarrow ((\neg r) \lor s)$

## Binding priority:

- An operand should be associated with the symbol of higher prion - If operand is surrounded by symbols with equal priority, then associate the operand to the right.
- Example :
- $\begin{array}{cccc}
  \mathbf{TP} & \mathbf{V} & \mathbf{Q} & \vdots & (\mathbf{TP}) & \mathbf{V} & \mathbf{Q} \\
  \mathbf{P} & \mathbf{V} & \mathbf{Q} & \mathbf{V} & \vdots & \mathbf{P} & \mathbf{V} & (\mathbf{Q} & \mathbf{V} & \mathbf{Y}) \\
  \end{array}$

- $p \vee q \wedge r : p \vee (q \wedge r)$ 
  - $\neg P \rightarrow q : (\neg p) \rightarrow q$
- $p \rightarrow q \rightarrow \lambda$  :  $p \rightarrow (q \rightarrow \lambda)$

More examples: Paranthesize the following formulas



- $(\gamma v (pr)) \leftarrow (qr) \gamma v pr \leftarrow qr$
- $p \wedge q \rightarrow r (p \wedge q) \rightarrow (r)$
- $p \wedge q \wedge n \rightarrow 8 (p \wedge (q \wedge s)) \rightarrow 8$
- $\neg p \rightarrow q \rightarrow n \wedge I \qquad (\neg q \rightarrow (\gamma \wedge s))$ 
  - 7p -> (q ->(n ns))

What we have seen so far?

- Propositional logic
- Syntax: constructing formulas from propositions

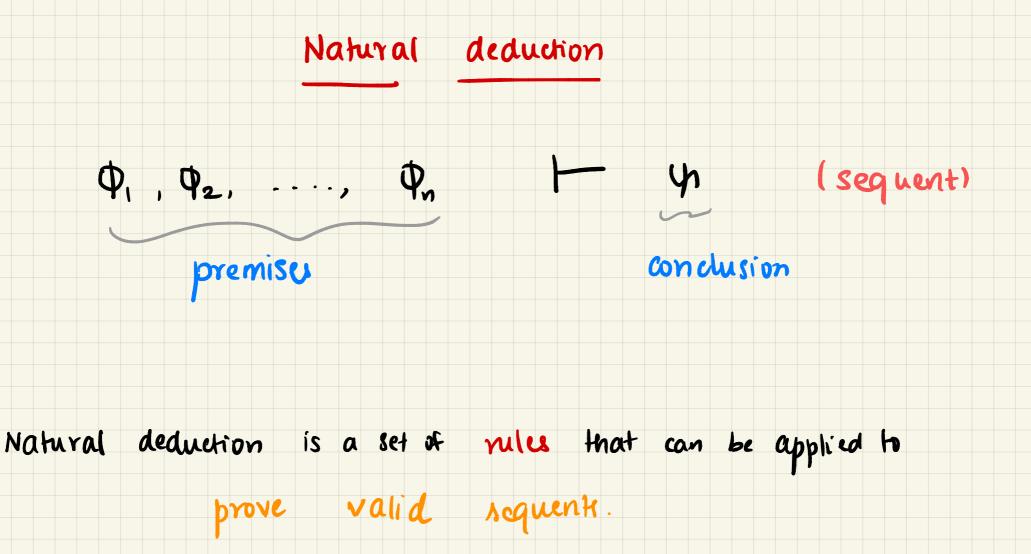
binding rules

- Semantics: meaning of propositional formulas

given by truth tables

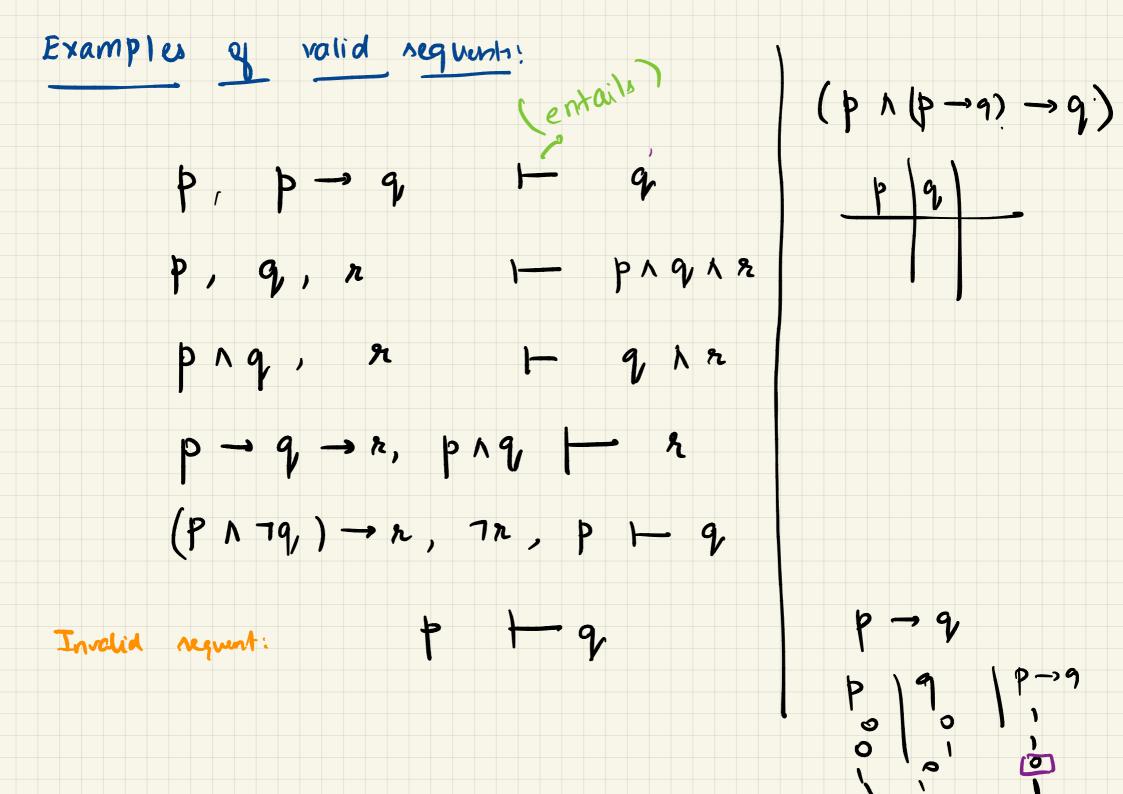
Next: a system of inference rules called

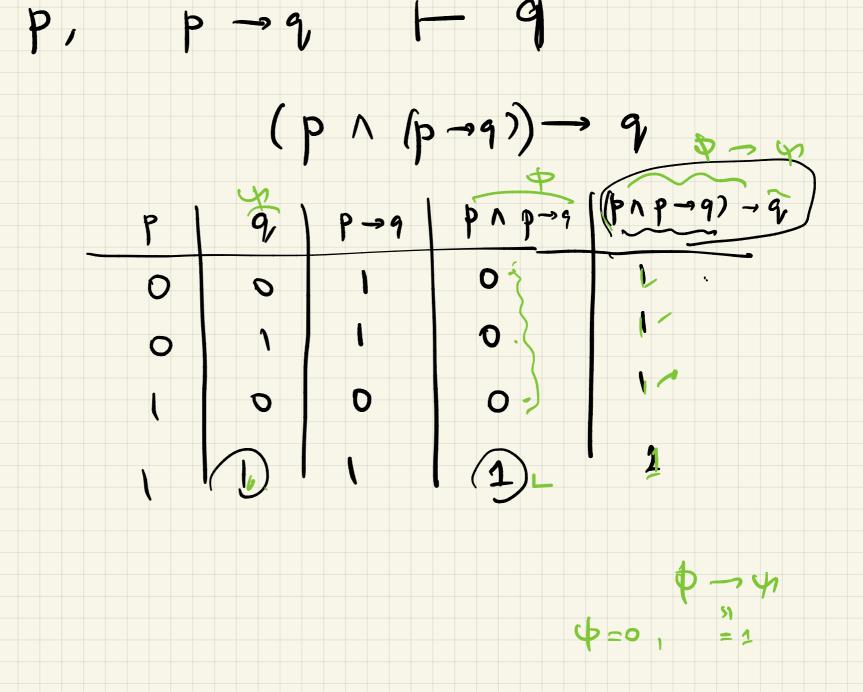
Natural Deduction



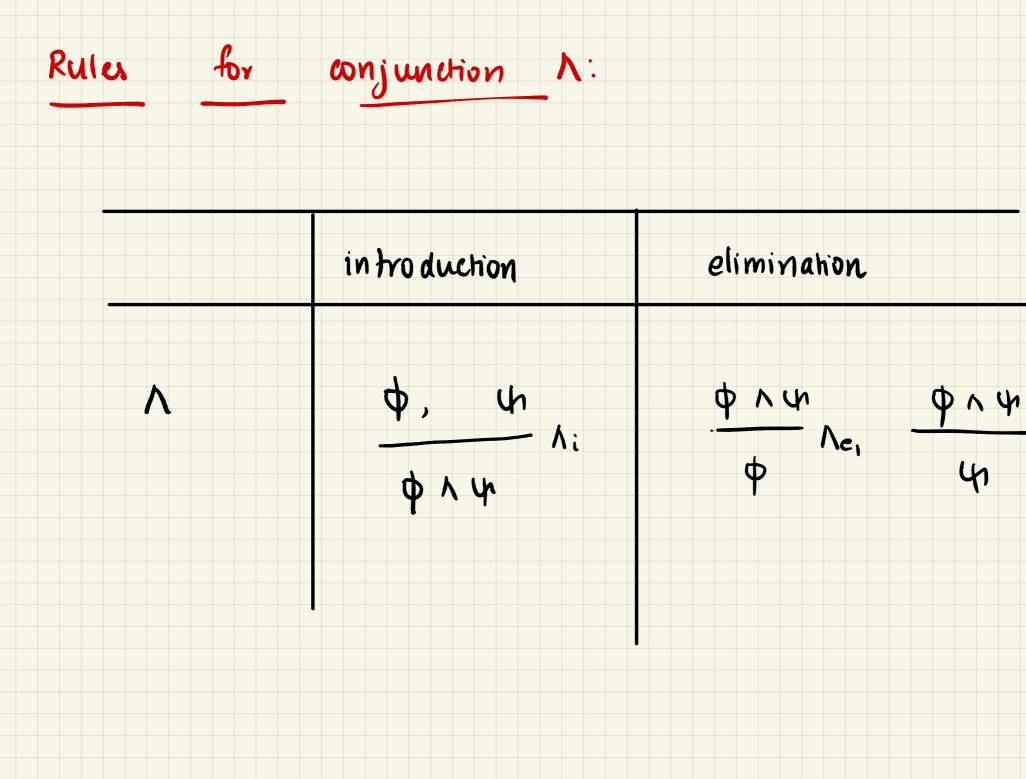
For valid sequents, the muth table of  $\phi_1 \land \phi_2 \land \dots \land \phi_n \twoheadrightarrow \phi_1$ 

has 1 in all the rows



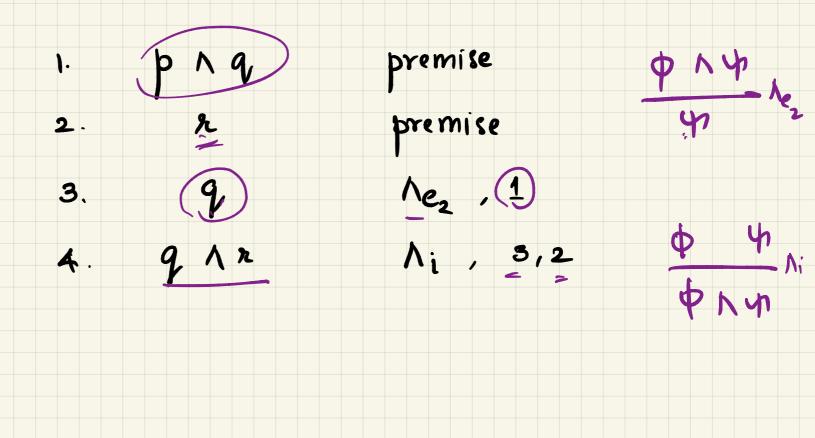


## Coming next Natural deduction rules

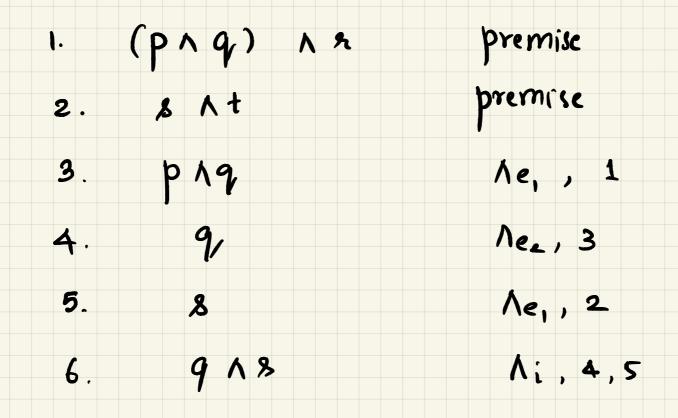


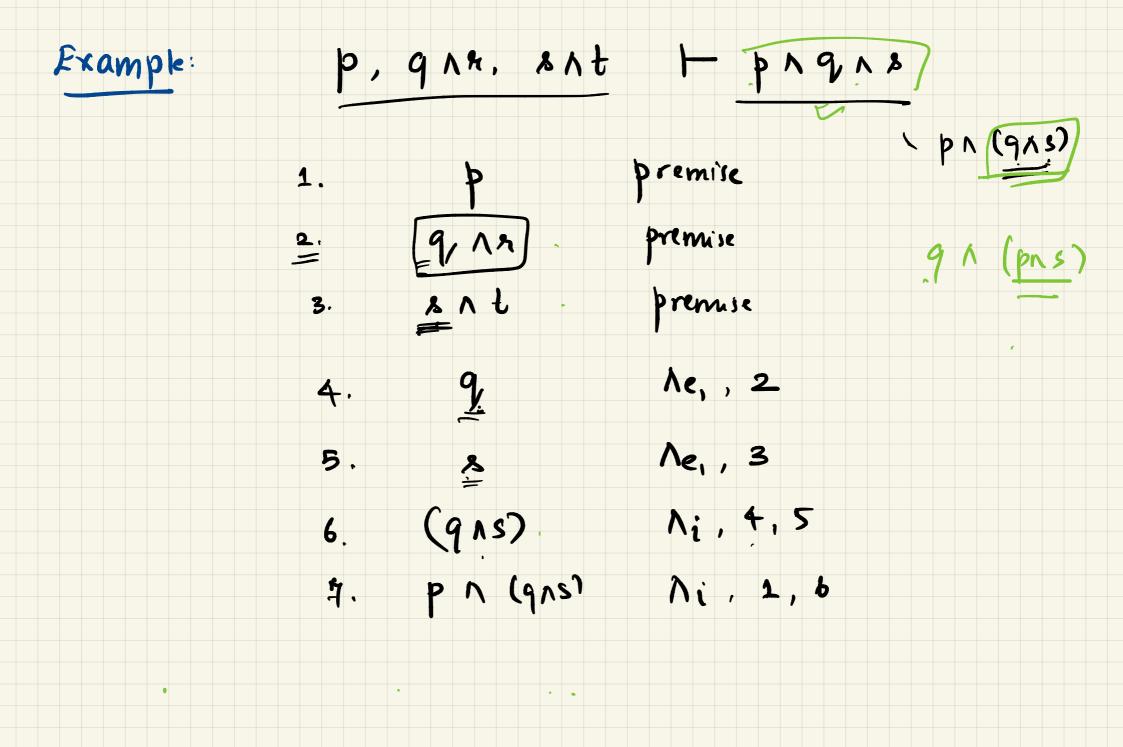
-Ne2

## Example: PAQ, n H- QAR



## Example: (prg) Nr, srt F grs







- -> Propositional logic (how to write tomulas)
- Truth tables for formular.
- Natural deduction.

-> write a proof using rules that shows this request is valid.