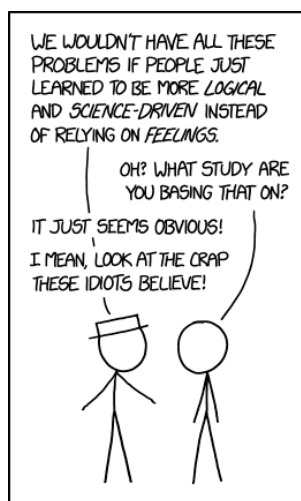# Problem Set 3
Discrete Mathematics 2019

CHENNAI MATHEMATICAL INSTITUTE

August 23, 2019

Problem 1. Determine whether each of these conditional statements is true or false.

a) If $1 + 1 = 3$, then next problem set will be easy.

b) If $1 + 1 = 3$, then there will be no end semester exams for this course.

c) If $1 + 1 = 3$, then next statement is true.

d) If $1 + 1 = 2$, then this statement is true.

e) If $2 + 2 = 4$, then $1 + 2 = 3$.

Problem 2. Construct a truth table for each of these compound propositions.

a) $p \rightarrow \neg p$

b) $p \leftrightarrow \neg p$

c) $p \oplus (p \vee q)$

d) $(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$

e) $(p \oplus q) \to (p \oplus \neg q)$

Problem 3. State the converse, contrapositive, and inverse of each of these conditional statements.

a) If teacher is not coming next week, there will be a quiz.

b) I come to class whenever there is going to be a quiz.

c) A positive integer is a prime only if it has no divisors other than 1 and itself.

Problem 4. Explain, without using a truth table, why $(p \vee \neg q) \wedge (q \vee \neg r) \wedge (r \vee \neg p)$ is true when p, q, and r have the same truth value and it is false otherwise.

Problem 5. Evaluate each of these expressions.

a) $11000 \wedge (01011 \vee 11011)$

b) $(01111 \wedge 10101) \vee 01000$

c) $(01010 \oplus 11011) \oplus 01000$

d) $(11011 \vee 01010) \wedge (10001 \vee 11011)$

Problem 6. Use $\neg, \to, \wedge$ and $\vee$ to express the following declarative sentences in propositional logic; in each case state what your respective propositional atoms p, q, etc. mean:

a) If Buckley's wife is sick, he will visit the hospital.

b) If someone inquires a receptionist, and she has the information, she will help.

c) The doctor will contact Buckley when his wife is sick and he is worried.

d) If the doctor is NOT on time then a child dies and the mother cries.

Problem 7. You are eligible to be President of the U.S.A. only if you are at least 35 years old, were born in the U.S.A, or at the time of your birth both of your parents were citizens, and you have lived at least 14 years in the country. Express your answer in terms of e: "You are eligible to be President of the U.S.A.," a: "You are at least 35 years old," b: "You were born in the U.S.A," p: "At the time of your birth, both of your parents where citizens," and r: "You have lived at least 14 years in the U.S.A."

WHY I TRY NOT TO BE
PEDANTIC ABOUT CONDITIONALS.
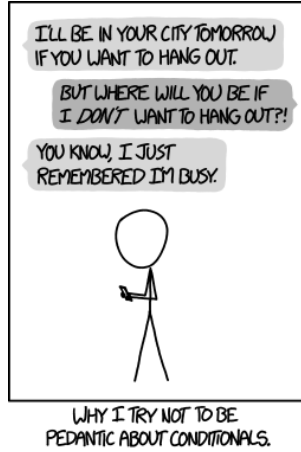
Problem 8. Prove the validity of the following sequents:

a) $(p \wedge q) \wedge r, s \wedge t \vdash q \wedge s$

b) $p \to (p \to q), p \vdash q$

c) $q \to (p \to r), \neg r, q \vdash \neg p$

d) $q \to r \vdash (p \to q) \to (p \to r)$

e) $p \vee q \vdash r \to (p \vee q) \wedge r$

f) $p \to q \vdash \neg q \to \neg p$

g) $p \to q, r \to \neg t, q \to r \vdash p \to \neg t$

h) $p \to q \vdash \neg p \vee q$

i) $\vdash \neg p \vee q \to (p \to q)$

j) $\vdash (p \to q) \vee (q \to r)$

Problem 9. For the sequents below, show which ones are valid and which ones aren't:

a) $\neg p \to \neg q \vdash q \to p$

b) $\neg p \vee \neg q \vdash \neg(p \wedge q)$

c) $p \vee q, \neg q \vee r \vdash p \vee r$

d) $p \wedge \neg p \vdash \neg(r \to q) \wedge (r \to q)$

e) $\neg(\neg p \vee q) \vdash p$

*Problem 10. The nth statement in a list of 100 statements is "Exactly n of the statements in this list are false."
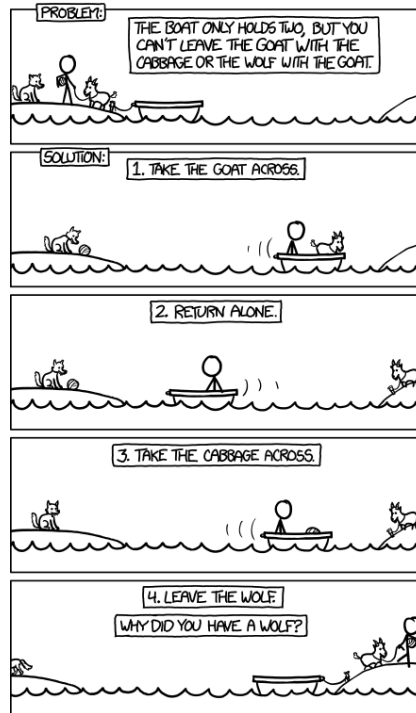
a) What conclusions can you draw from these statements?

b) Answer part (a) if the nth statement is "At least n of the statements in this list are false."

c) Answer part (b) assuming that the list contains 99 statements.

*Problem 11. Sudoku is a placement puzzle. The aim of the puzzle is to enter a numeral from1 through 9 in each cell of a grid, most frequently a 9×9 grid made up of 3×3 subgrids (called "regions"), starting with various numerals given in some cells(the "givens"). Each row, column and region must contain only one instance of each numeral. Its grid layout is like the one shown in the following schema:

6					3			5
9		3		8				
	5	1					6	
			4	3				7
		8	5		7	1		
4				6	8			
		7				9	8	
				7		4		2
8			3					6

Provide a formalization in propositional logic of the Sudoku problem, so that any truth assignment to the propositional variables that satisfy the axioms isa solution for the puzzle.

Hint: For $1 \leq n, r, c \leq 9$, define the proposition $in(n, r, c)$ which means that the number n has been inserted in the cross between row r and column c.