Playing with repeating values in datawords using energy games

Diego Figueira and M. Praveen

Outline

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Specifications for a coffee machine

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The realizability problem:
Input: A formula, a partition of the variables
Question: Does the system have a winning strategy?

Outline

Model, syntax, semantics

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\begin{array}{lccccccc}
x: & d_1 & \$ & * & \cdots & * & * & \cdots \\
y: & d_2 & * & \$ & \cdots & \# & \$ & \cdots \\
w: & \top & \bot & \top & \cdots & * & * & \cdots
\end{array}
$$

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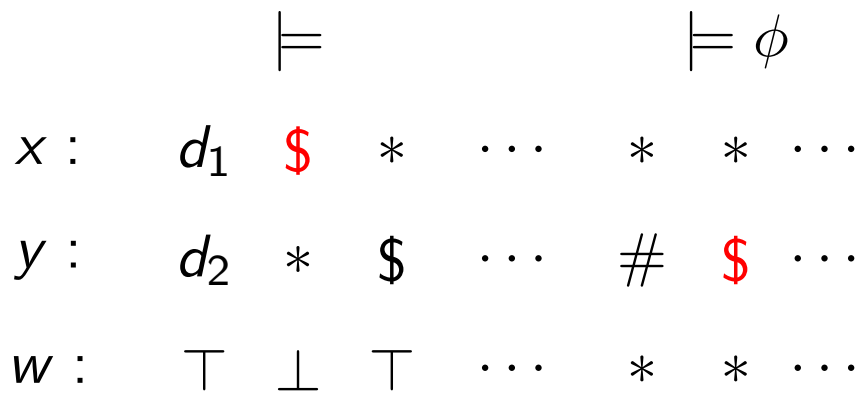

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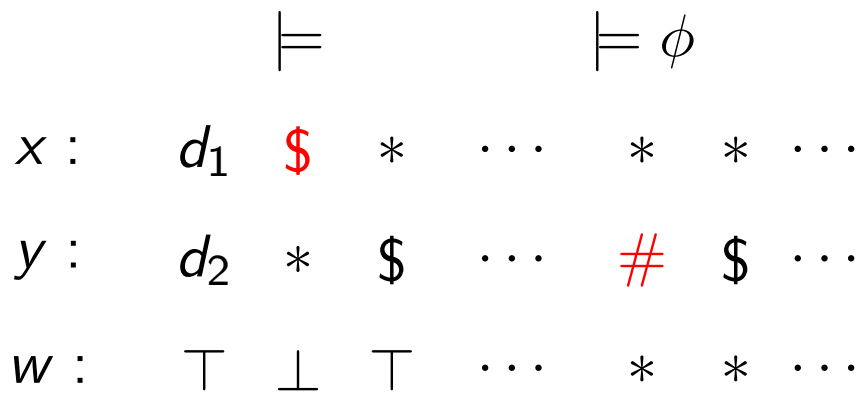

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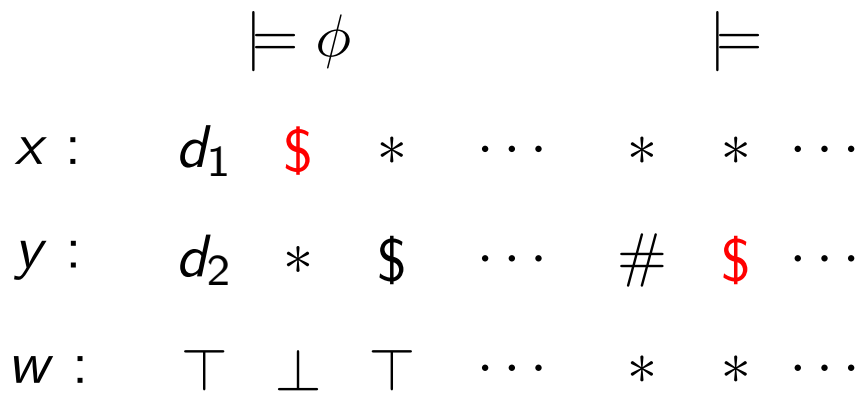

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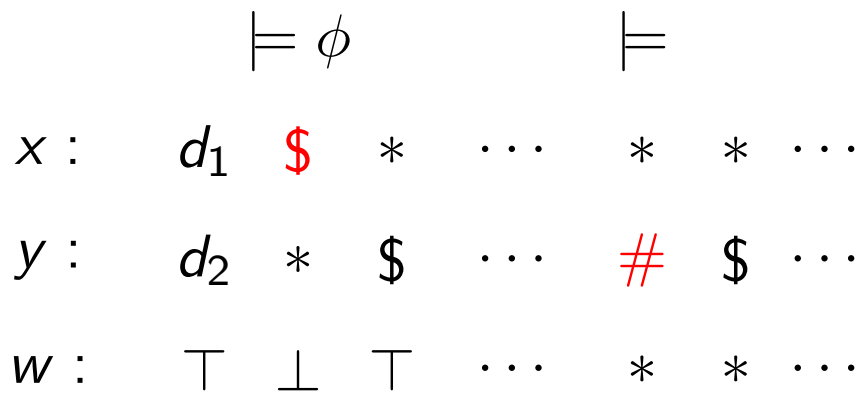

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- Realizability of LRV: parity games on VASS.

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Asymmetry in games on VASS

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- [Bérard, Haddad, Sassolas, Sznajder 2012] One palyer can only increment; the other player cannot test for zero.

Asymmetry in games on VASS

- [Chatterjee, Randour, Raskin 2013] Energy games: if a player makes a counter to go below zero, the other player wins immediately. One of the players has to additionally satisfy a parity condition.

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- [Abdulla, Mayr, Sangnier, Sproston 2013] Single-sided VASS games: transitions that make some counter to go below zero are disabled for both players. One of the players cannot change counters; the other player has to additionally satisfy a parity condition.

Single-sided LRV games

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- Realizability can be reduced to single-sided VASS games.

Single-sided LRV games — symbolic models

Concrete model

$$\models y \approx \Diamond^{-1} x$$

$$\downarrow$$

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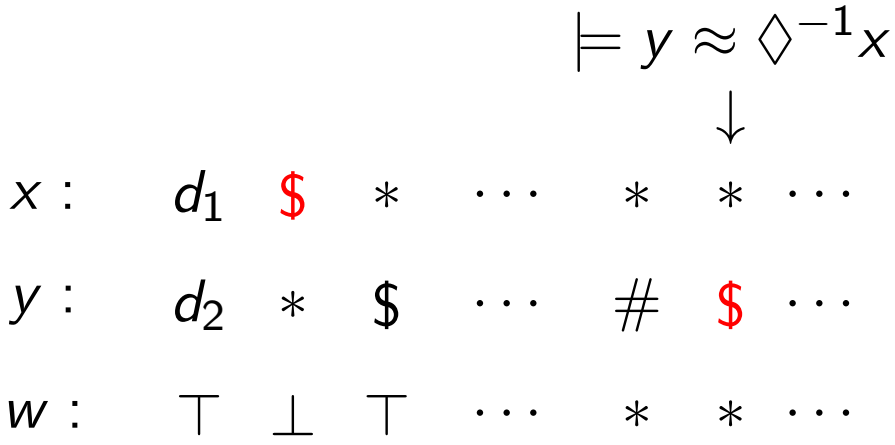

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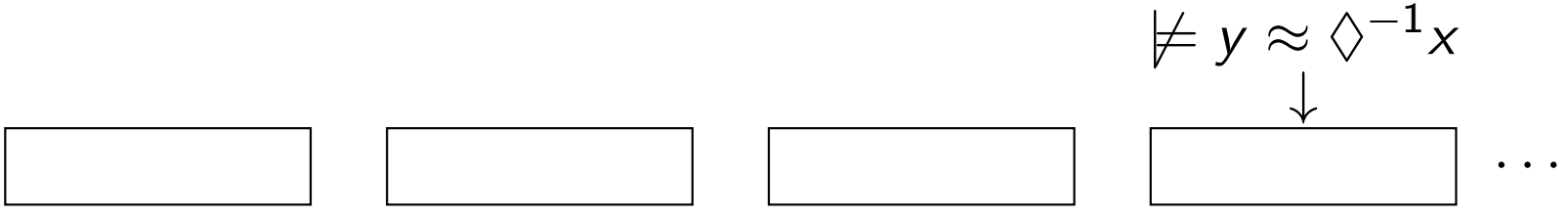

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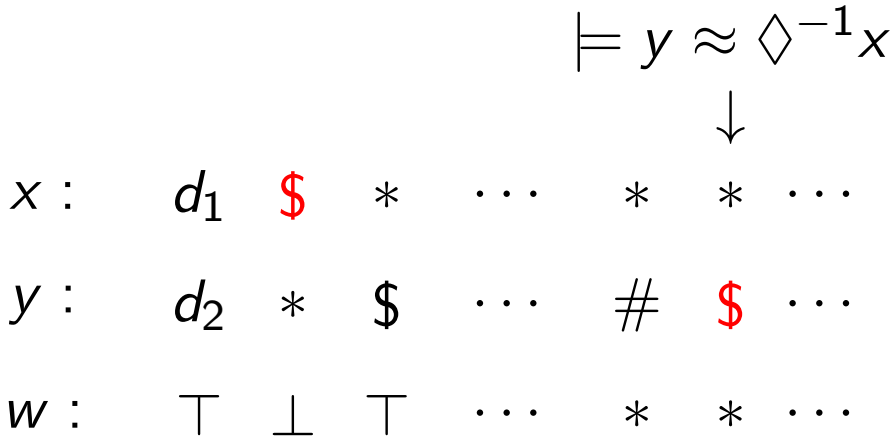

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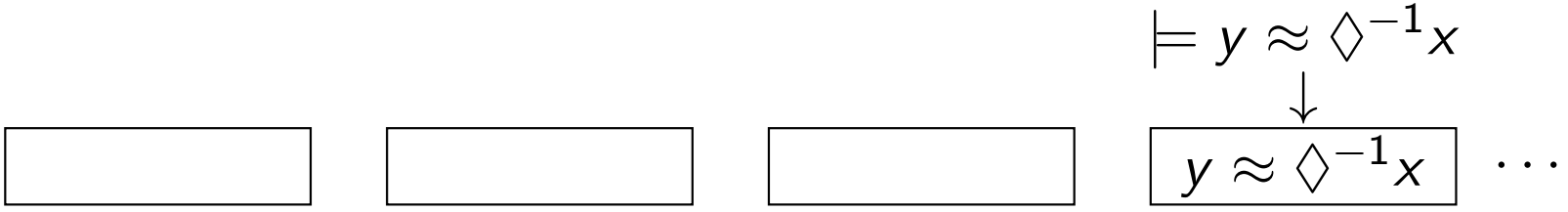

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$$\boxed{x \approx y} \quad \boxed{x \approx y} \quad \boxed{x \approx y} \quad \cdots \quad \boxed{x \approx y} \quad \begin{matrix} w \approx \Diamond^{-1} y \\ \boxed{w \approx \Diamond^{-1} x} \end{matrix}$$

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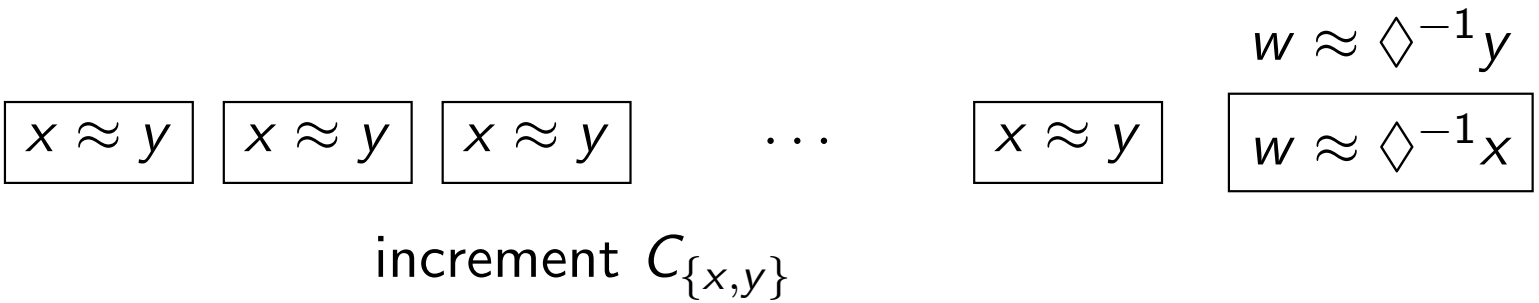

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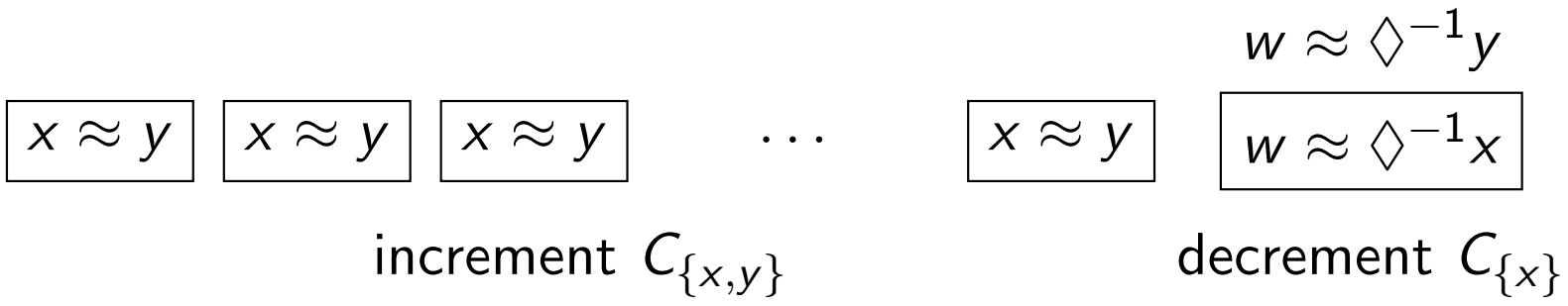

Outline

Restrictions to get decidability

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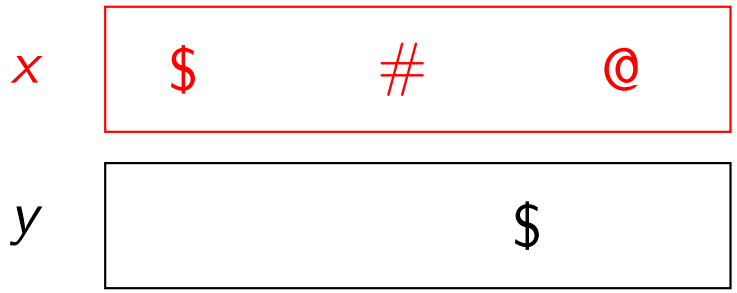
Simulating counter machines



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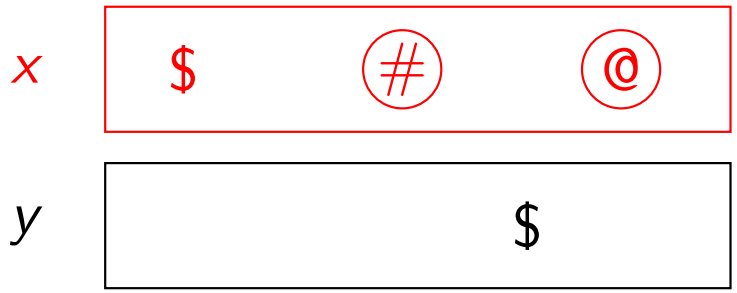

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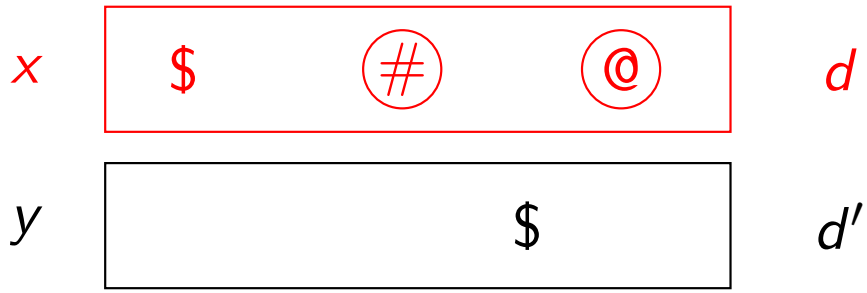
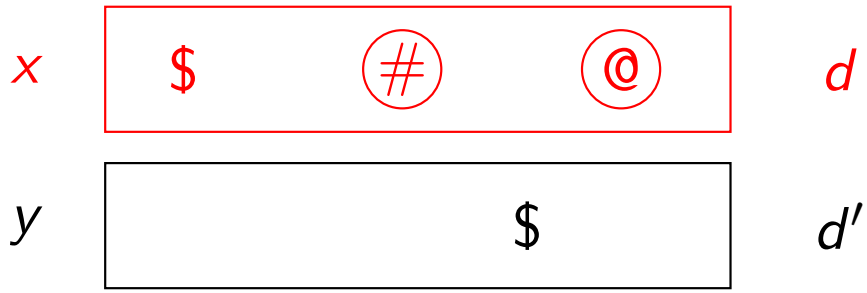


Increment

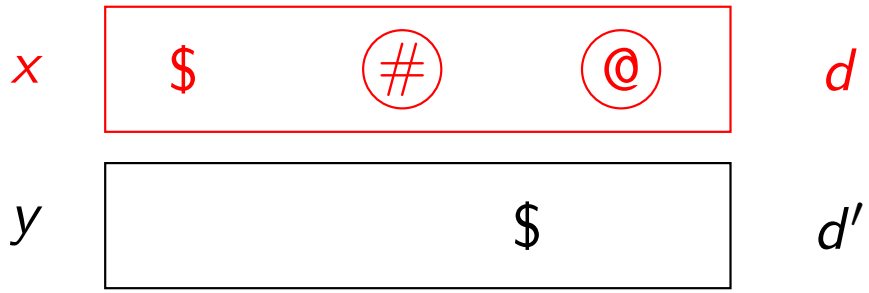
Simulating counter machines



Increment

- d doesn't repeat in the past in x or y

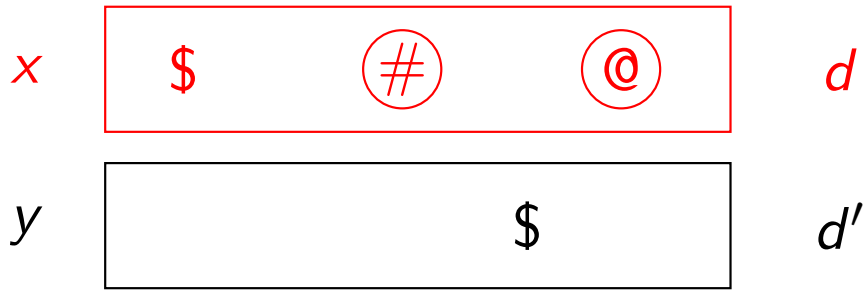
Simulating counter machines



Increment

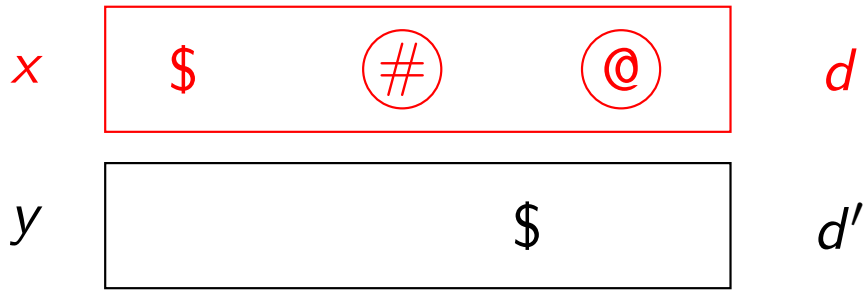
- d doesn't repeat in the past in x or y
- d' appears in the past in x and y

Simulating counter machines



Decrement

Simulating counter machines



Decrement

- d should appear in the past in x but not in y

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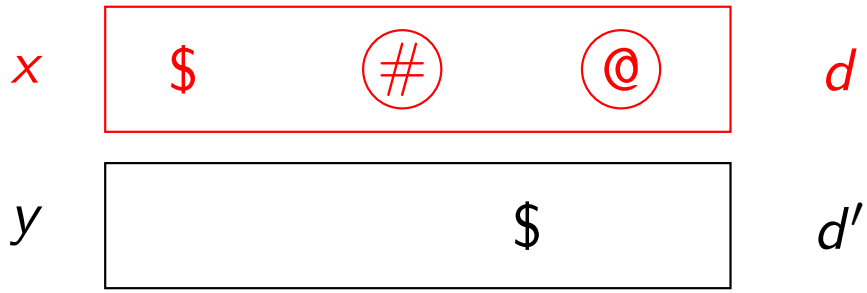

Decrement

- d should appear in the past in x but not in y
- $d' = d$

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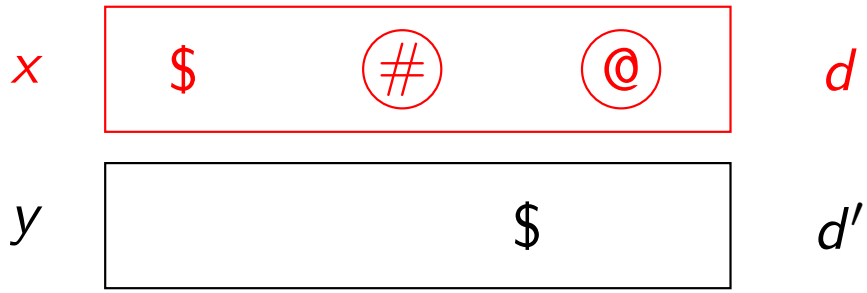
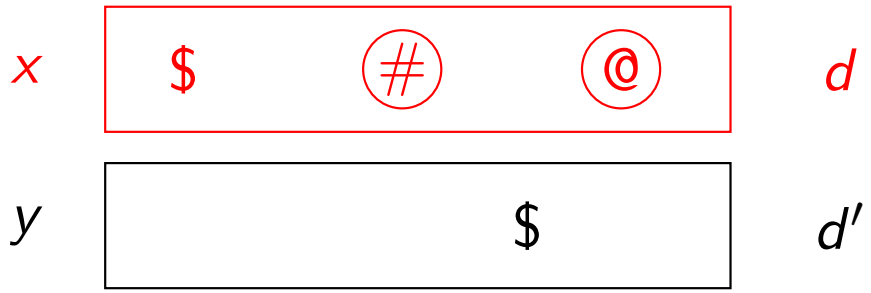


Zero test

Simulating counter machines



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- In case of no cheating, $d = d'$ and no repetition in the past.

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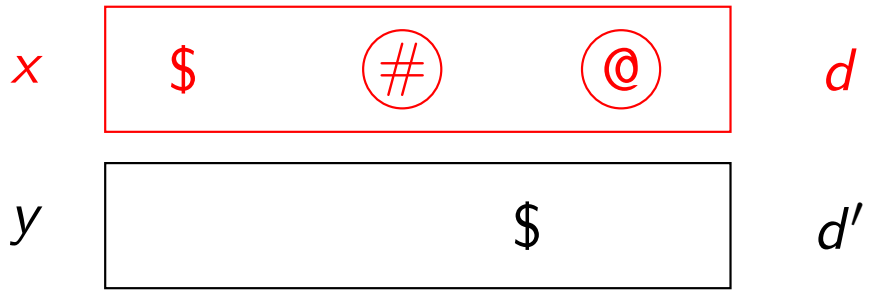

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- In case of cheating, second player can win immediately by ensuring d' repeats in the past in x but not in y.

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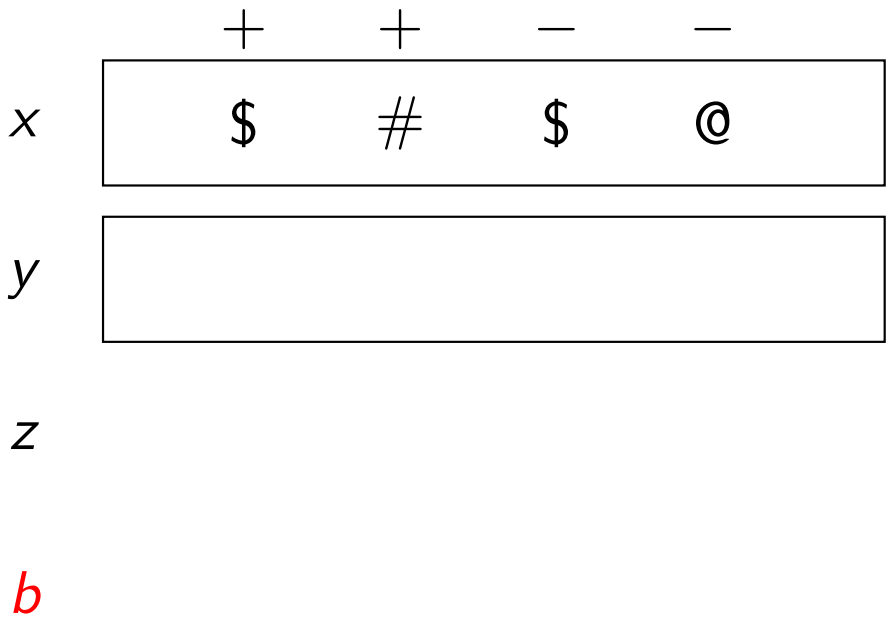

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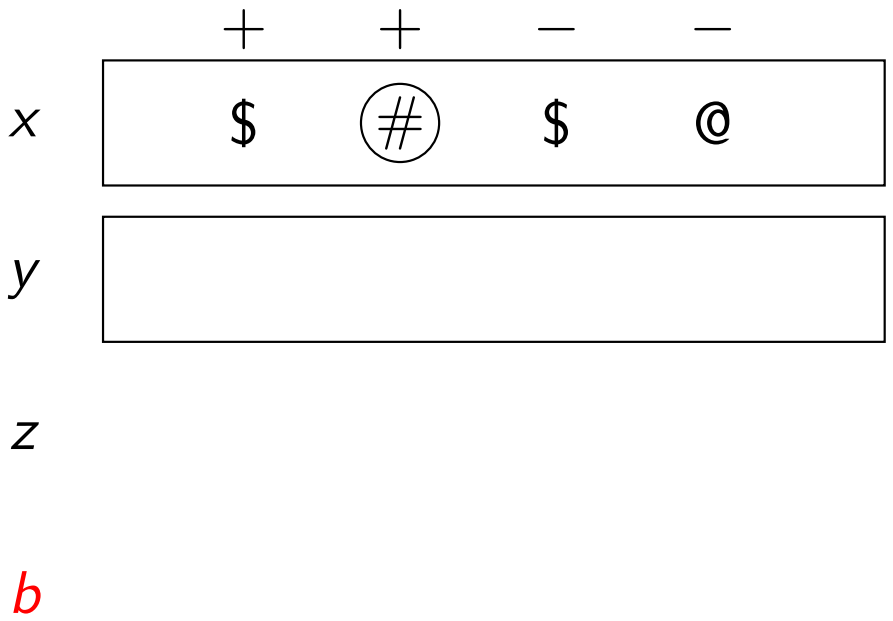

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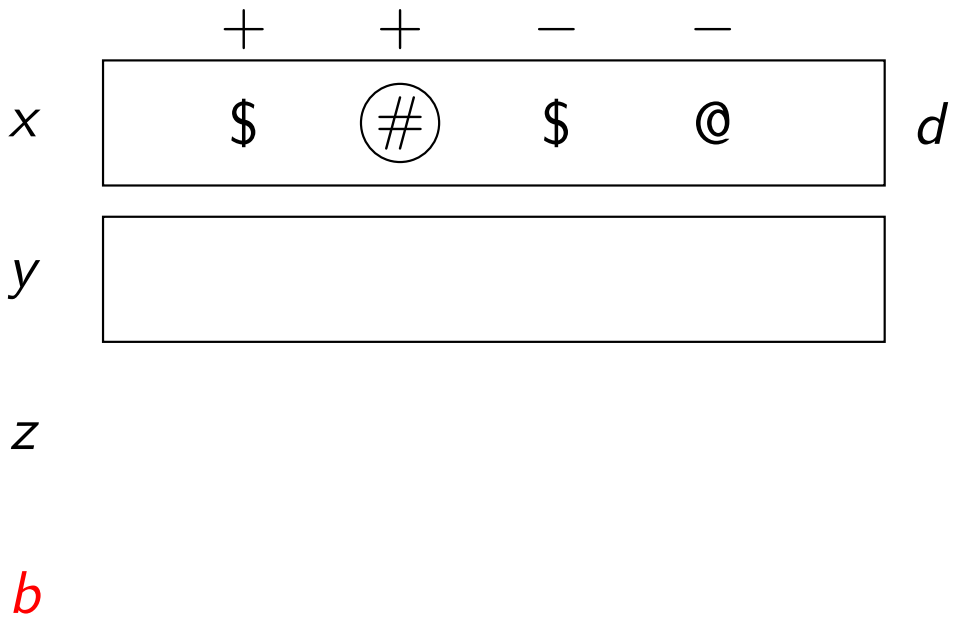

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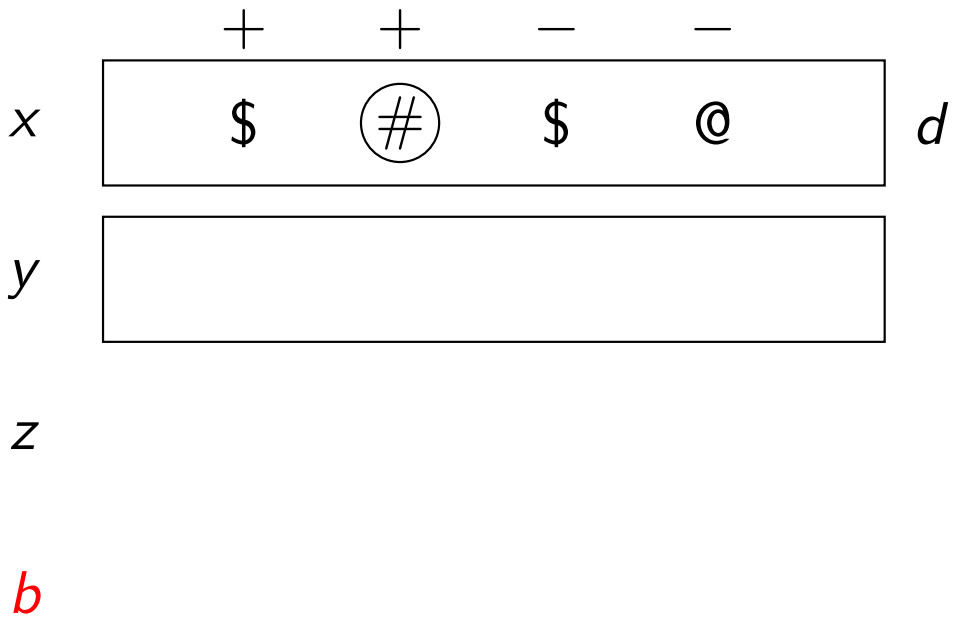

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- d should be a new data value.

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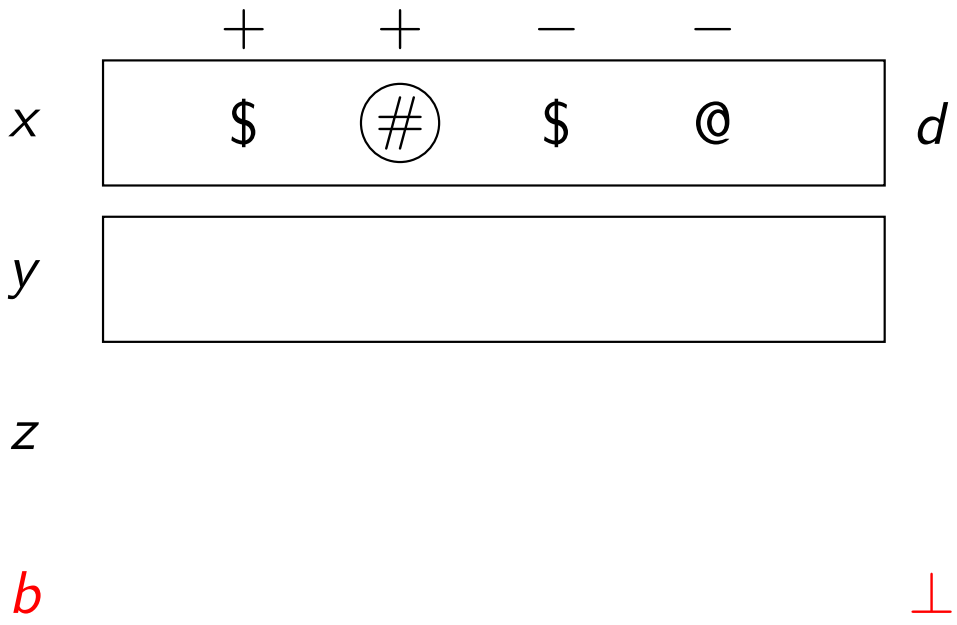
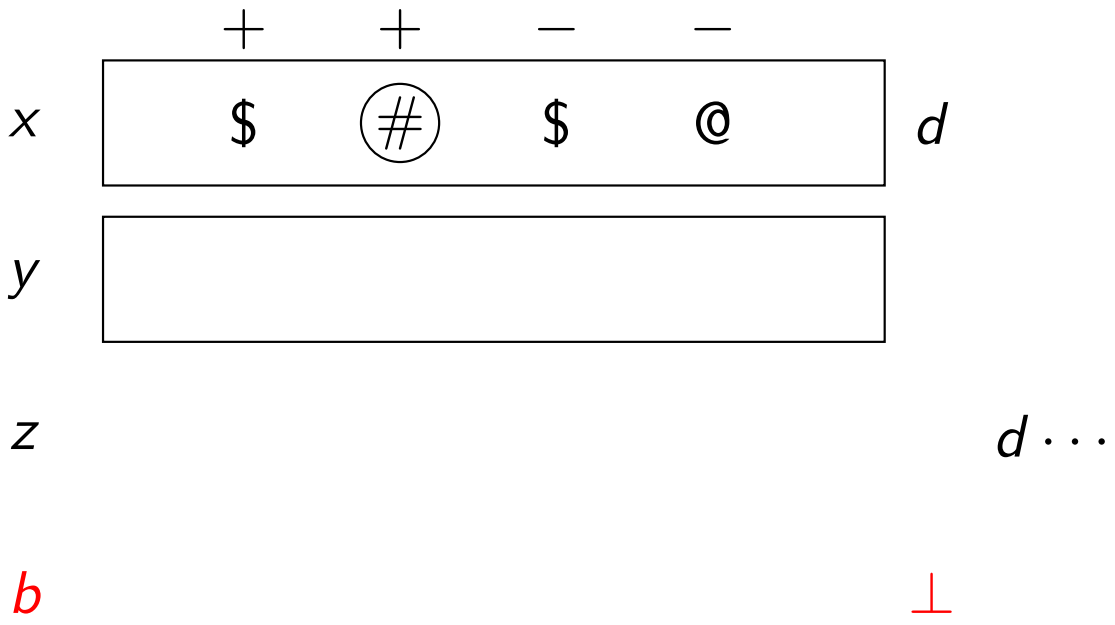


Increment

- d should be a new data value.
- If not, second player can set b to false.

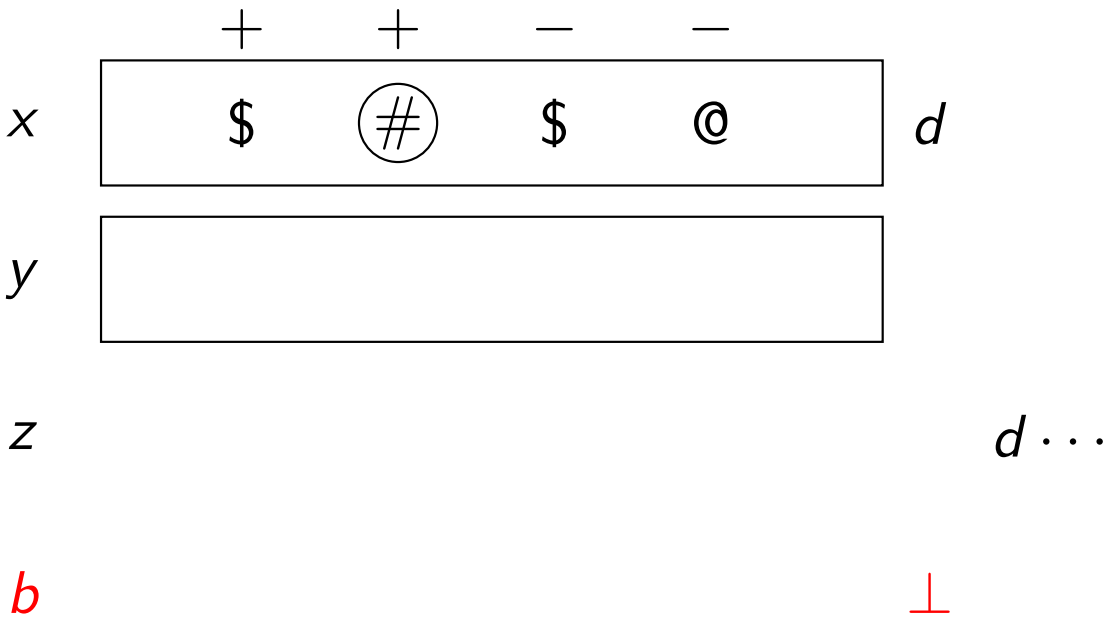
Simulating counter machines



Increment

- d should be a new data value.
- If not, second player can set b to false.
- First player must repeat d in z from next round.

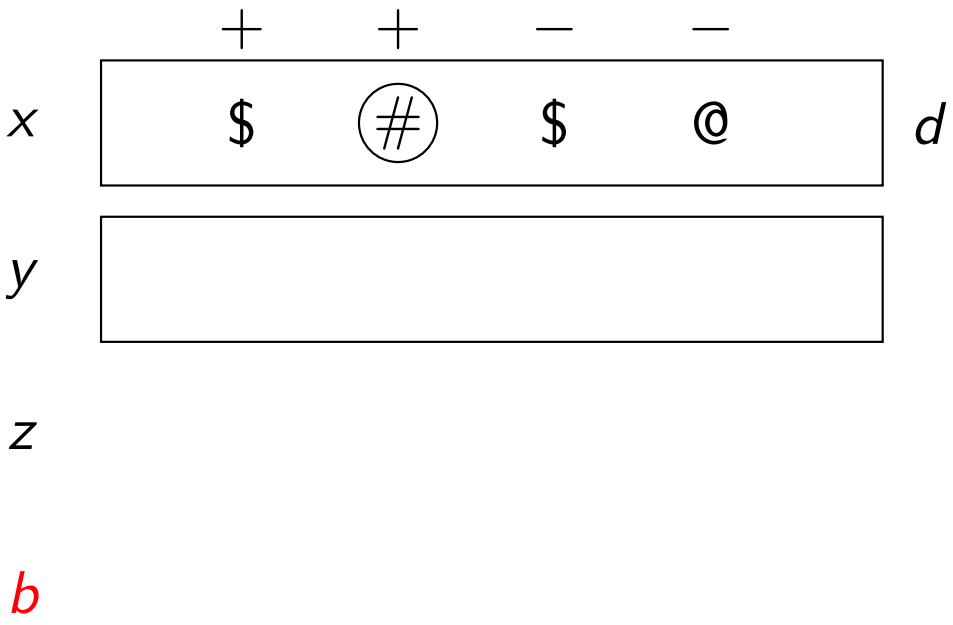
Simulating counter machines



Increment

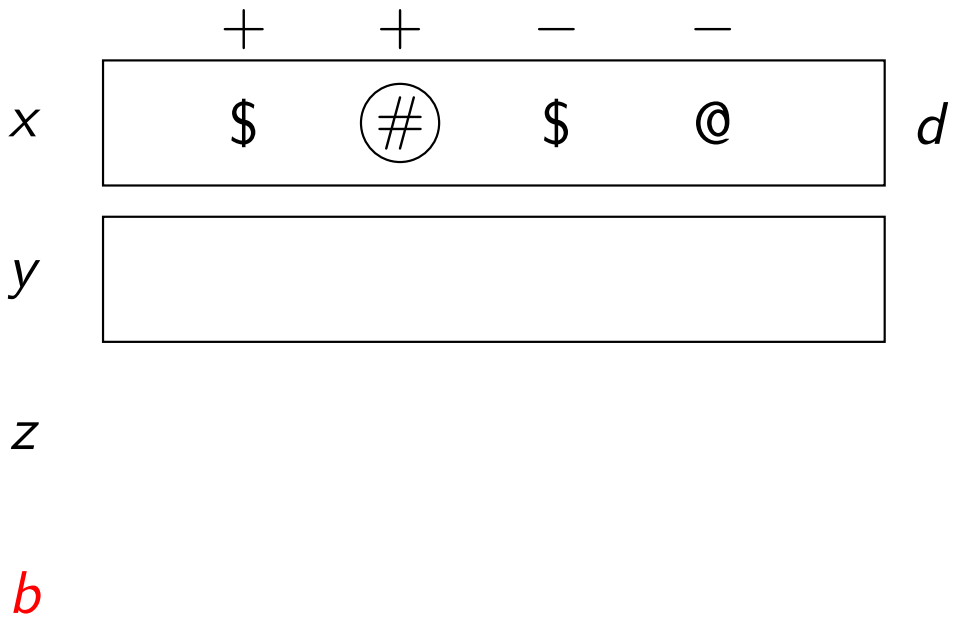
- d should be a new data value.
- If not, second player can set b to false.
- First player must repeat d in z from next round.
- If cheating on increment, $G(b \Rightarrow \neg(x \approx \Diamond z))$

Simulating counter machines



Decrement

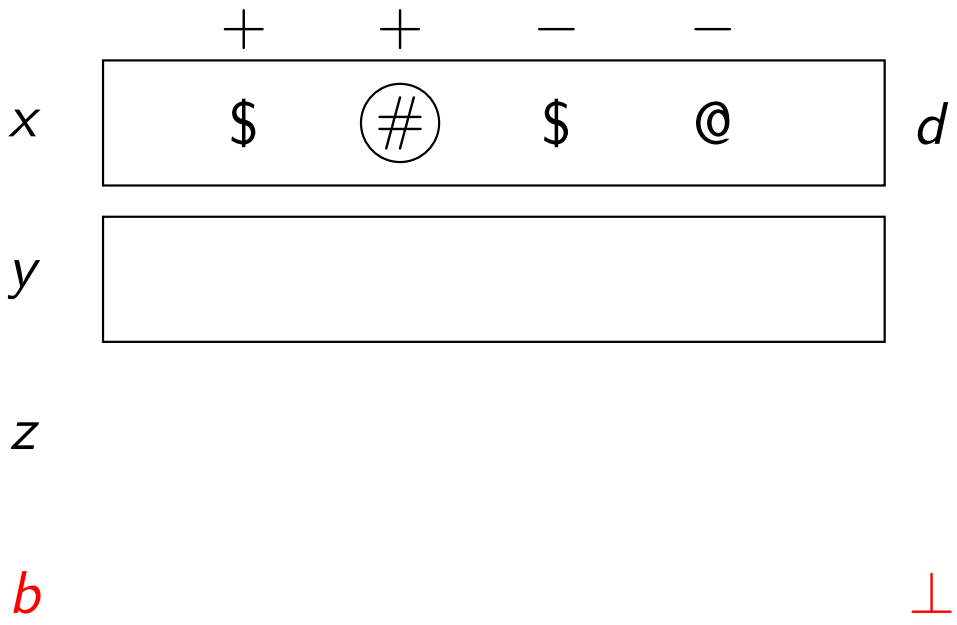
Simulating counter machines



Decrement

- d must repeat in the past in an incrementing position.

Simulating counter machines



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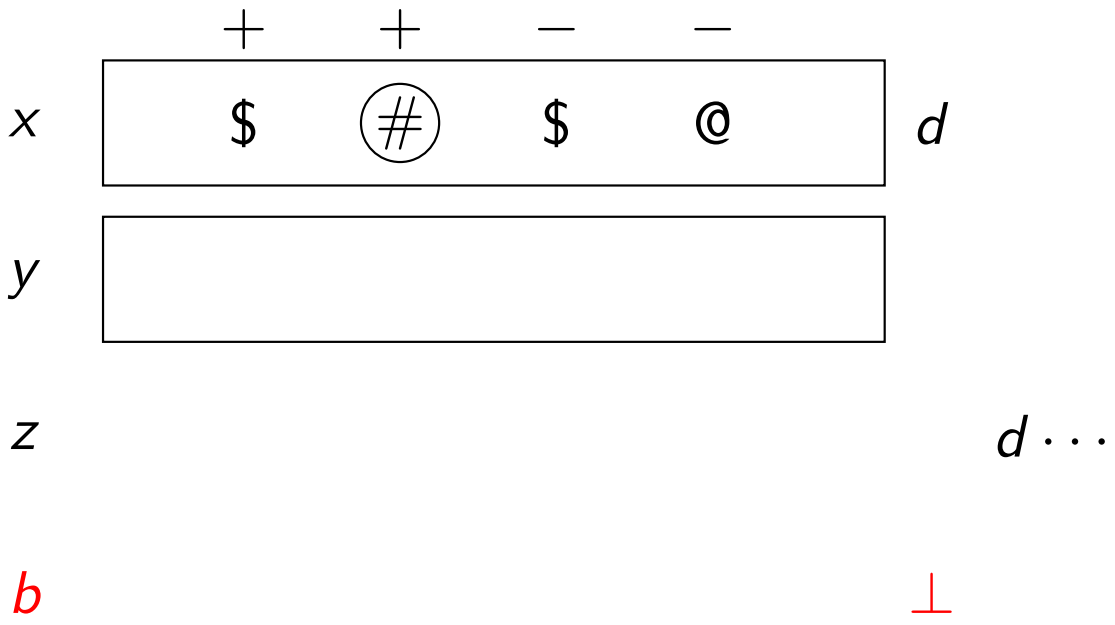
Simulating counter machines



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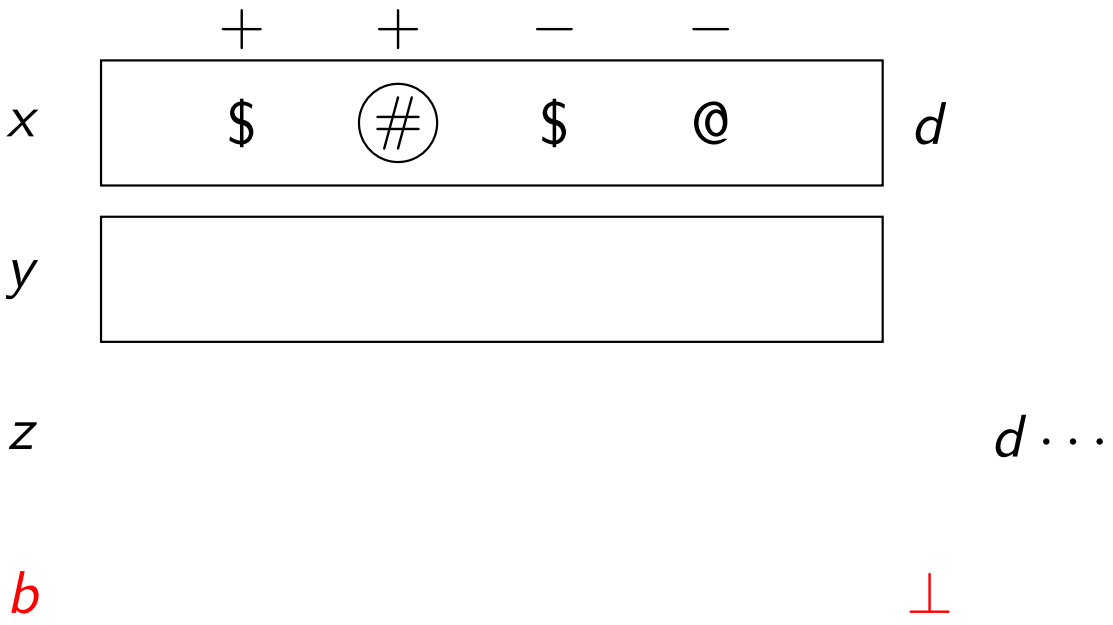
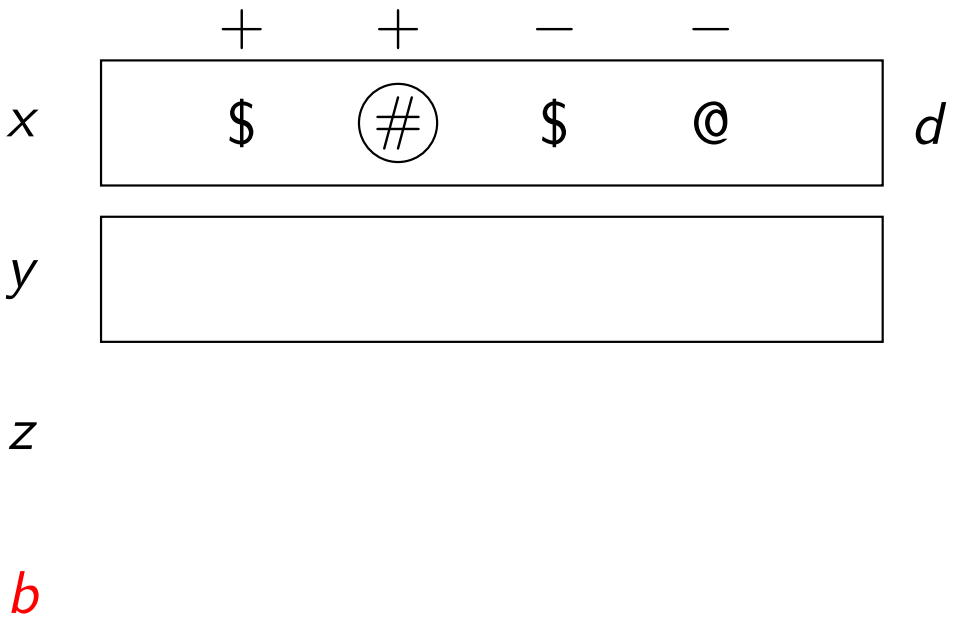
Simulating counter machines



Decrement

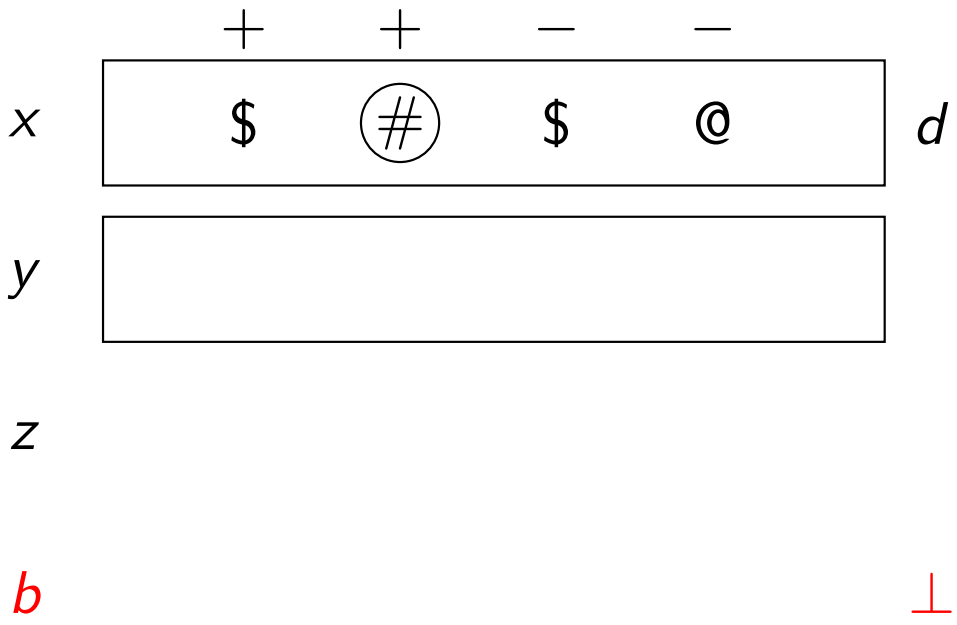
- d must repeat in the past in an incrementing position.
- If not, second player can set b to false.
- First player must repeat d in z from next round.
- If cheating on decrement, $F(b \wedge x \approx \Diamond z)$.

Simulating counter machines



Zero test

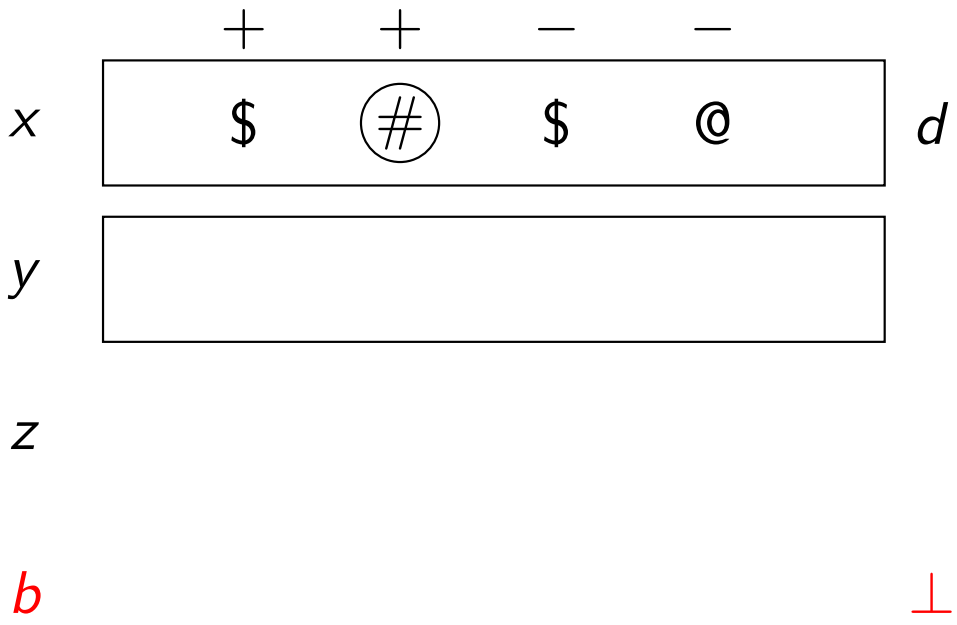
Simulating counter machines



Zero test

- Increment, no matching decrement $\Rightarrow$ b set to false.

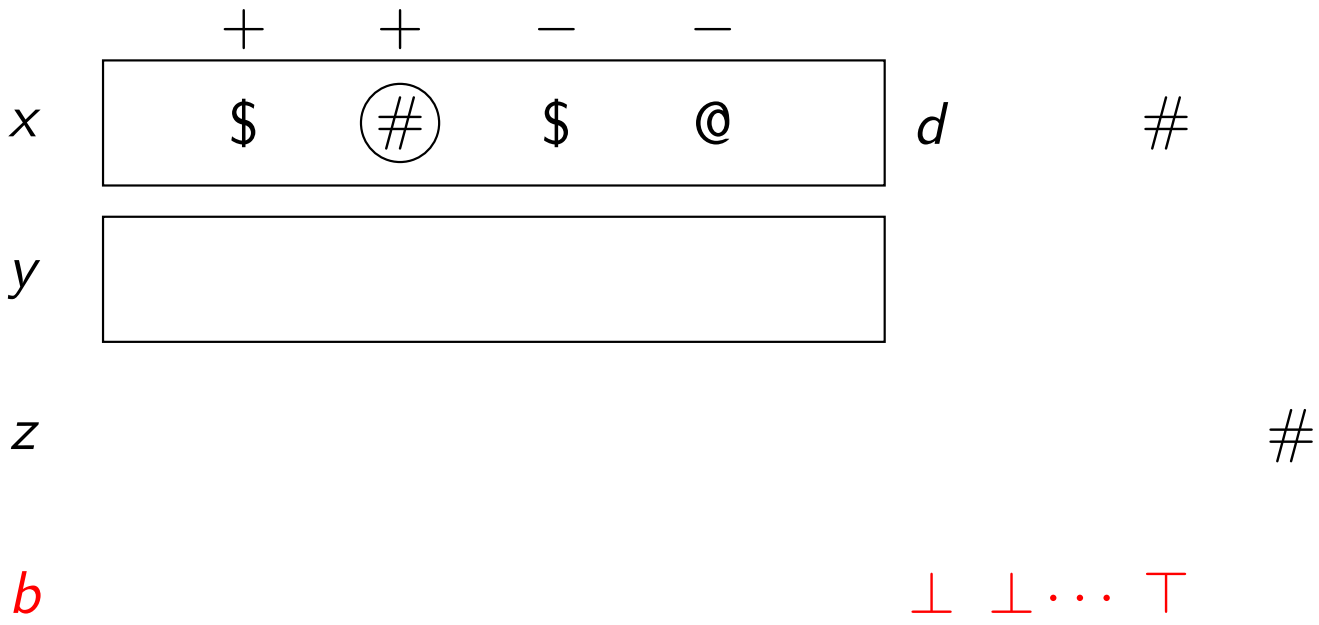
Simulating counter machines



Zero test

- Increment, no matching decrement $\Rightarrow$ b set to false.
- Increment, future zero test $\Rightarrow$ x must repeat in x.

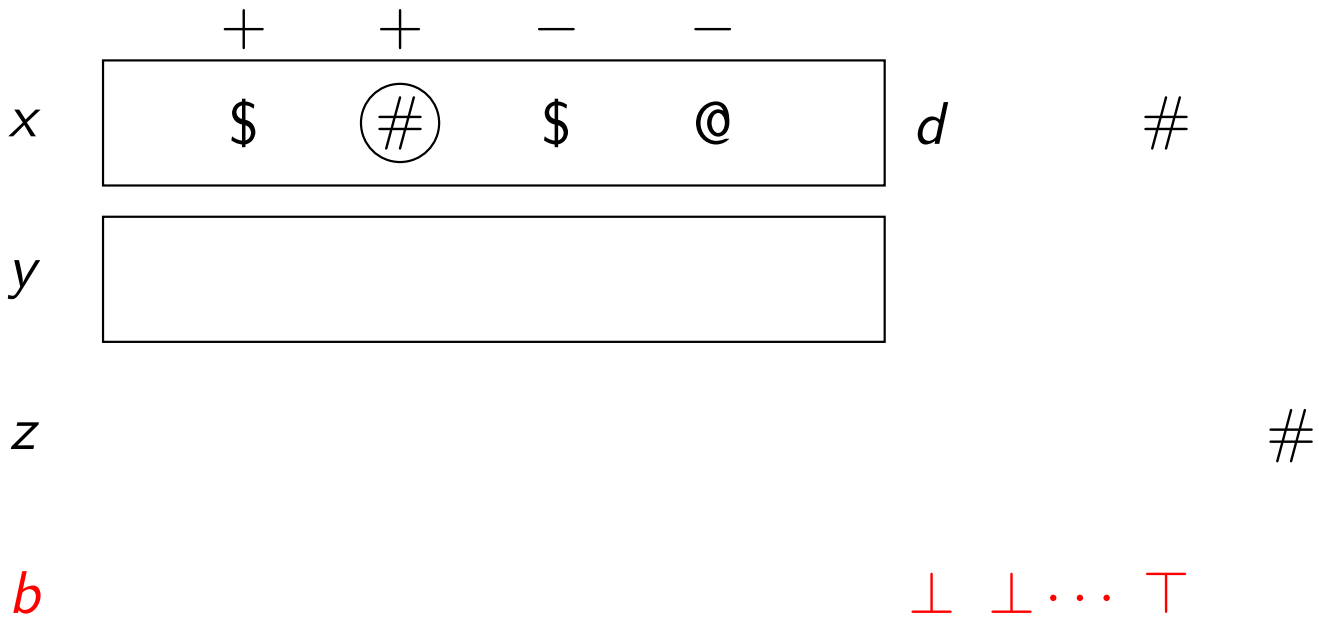
Simulating counter machines



Zero test

- Increment, no matching decrement $\Rightarrow$ b set to false.
- Increment, future zero test $\Rightarrow$ x must repeat in x.
- Upon repetition, b is set to true again; first player must repeat $\#$ in z.

Simulating counter machines



Zero test

- Increment, no matching decrement $\Rightarrow$ b set to false.
- Increment, future zero test $\Rightarrow$ x must repeat in x.
- Upon repetition, b is set to true again; first player must repeat $\#$ in z.
- If cheating on zero test Globally (b is true and incrementing $\Rightarrow$ x doesn't repeat in z).

Outline

Restrictions to get decidability

- No nested formulas: only $x \approx \langle \top ? \rangle^{-1} y$.
- Environment player has only Boolean variables.
- No future obligations: $x \approx \langle \top ? \rangle y$ not allowed.

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Thank you