Algorithmic Challenges in Radiation Therapy

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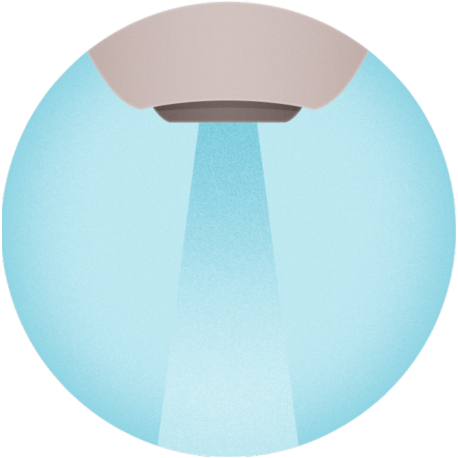
January, 2019

Complexity, Algorithms, Automata and Logic Meet 2019

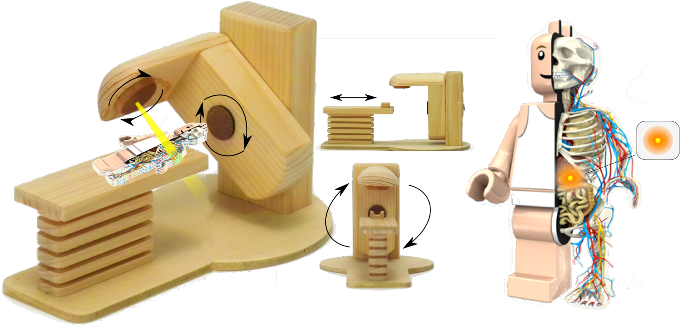
Radiation Therapy

Radiation Therapy

Cancer treatment relying on radiations aiming at killing cancerous cells.



Therapy modalities



External vs Internal

External beam therapy

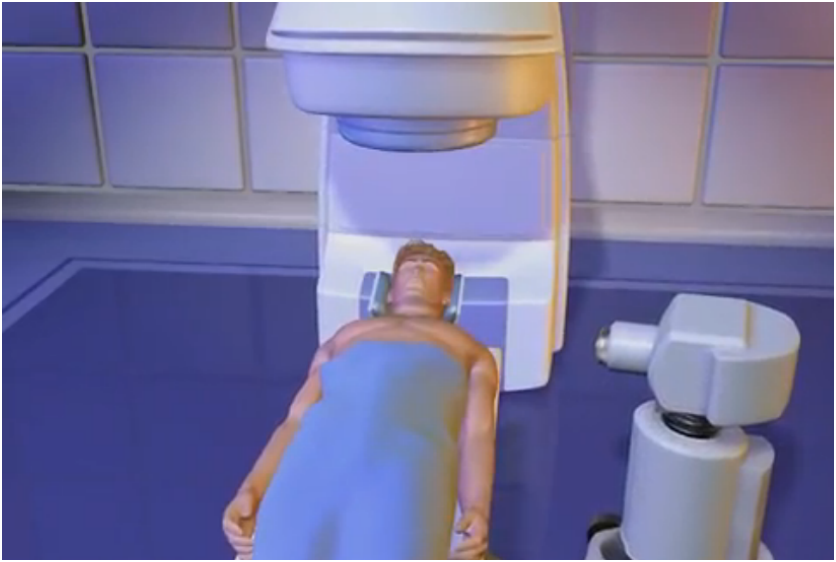
External beam therapy

External beam therapy

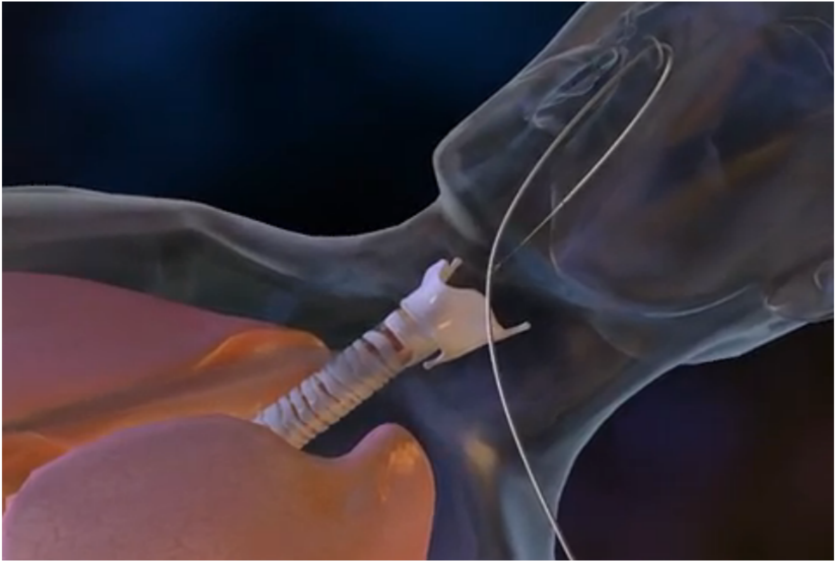
External beam therapy

External beam therapy

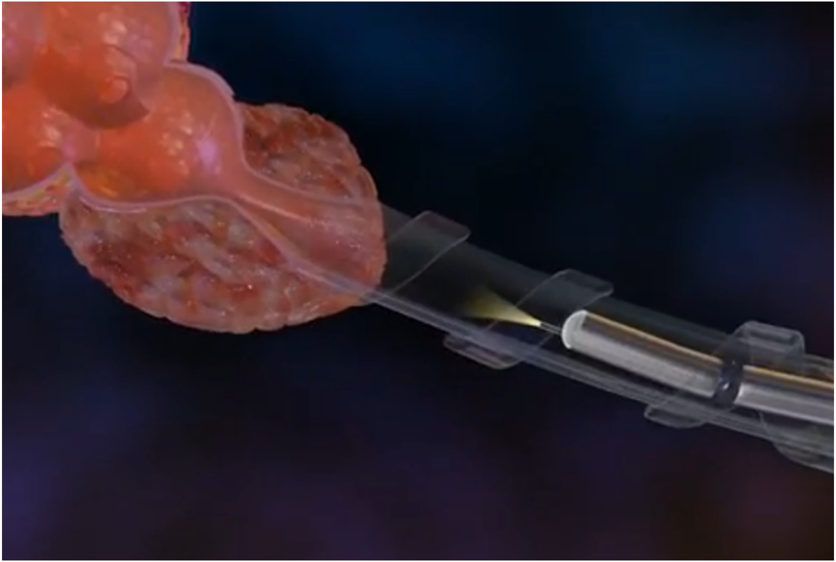
Internal beam therapy



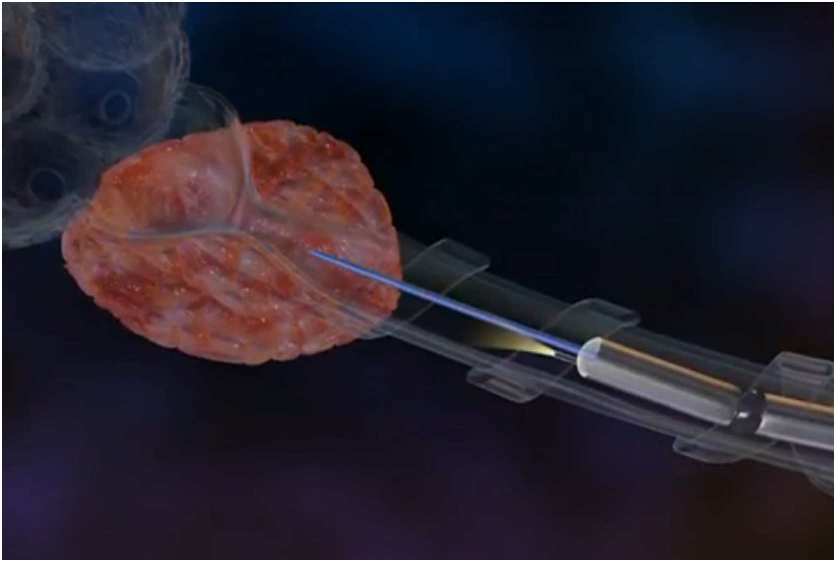
Internal beam therapy



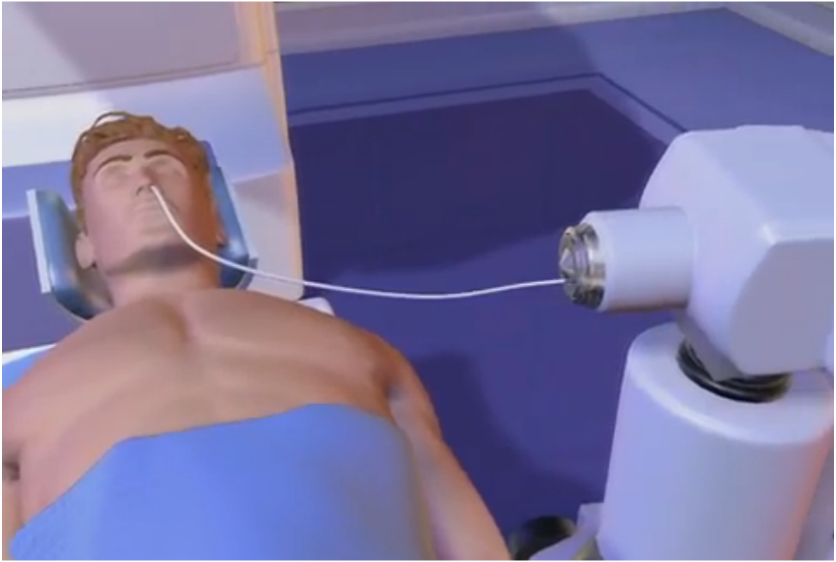
Internal beam therapy



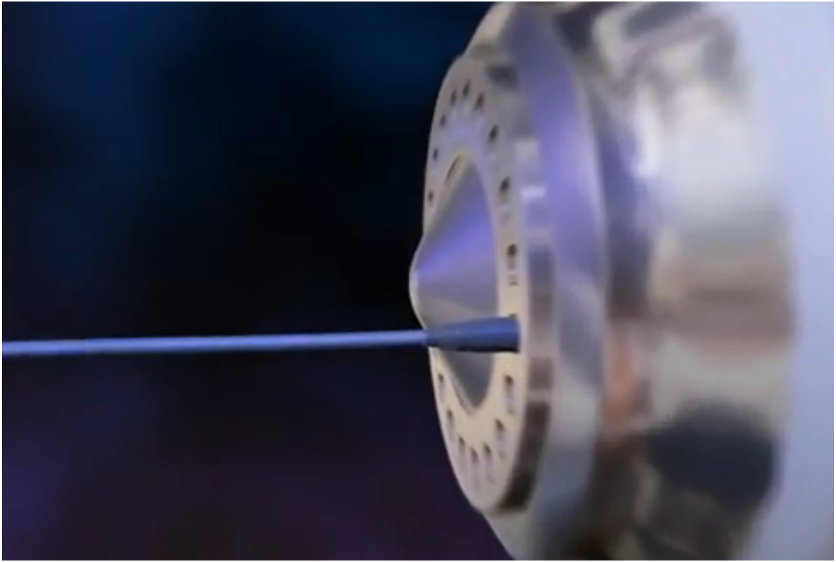
Internal beam therapy



Internal beam therapy



Internal beam therapy



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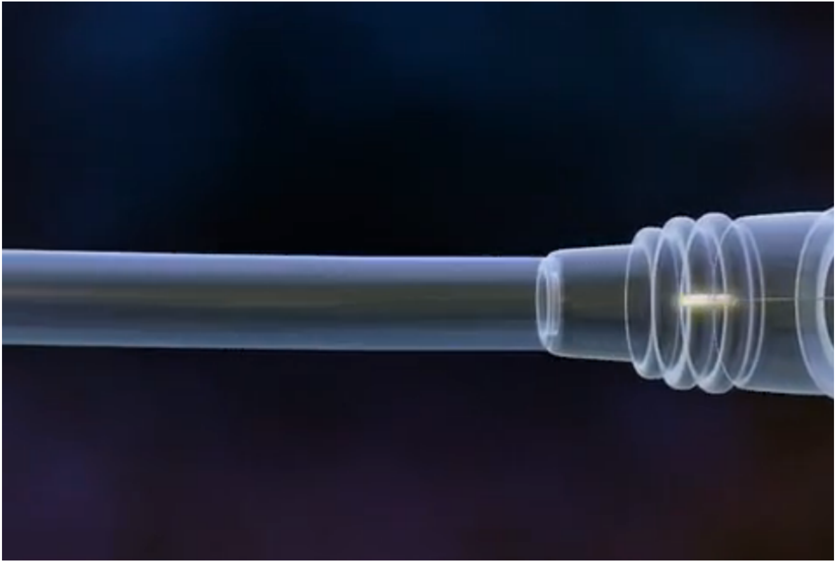
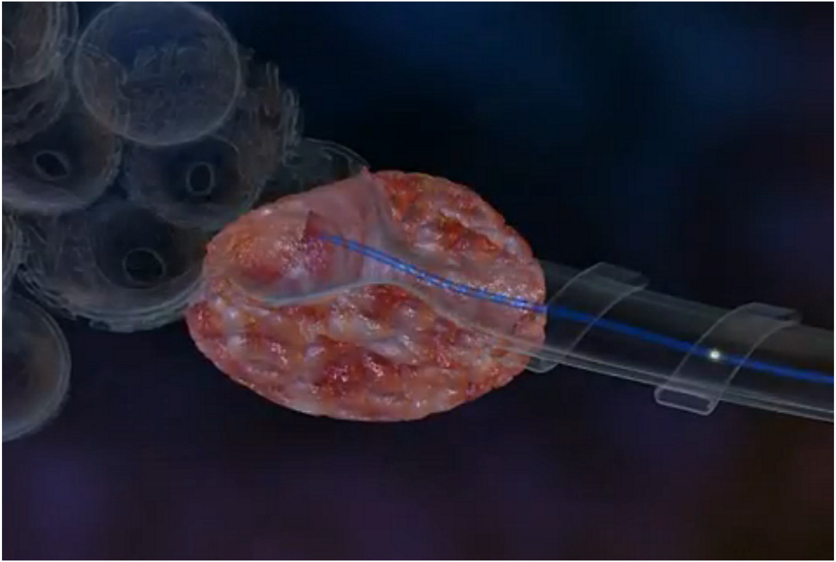
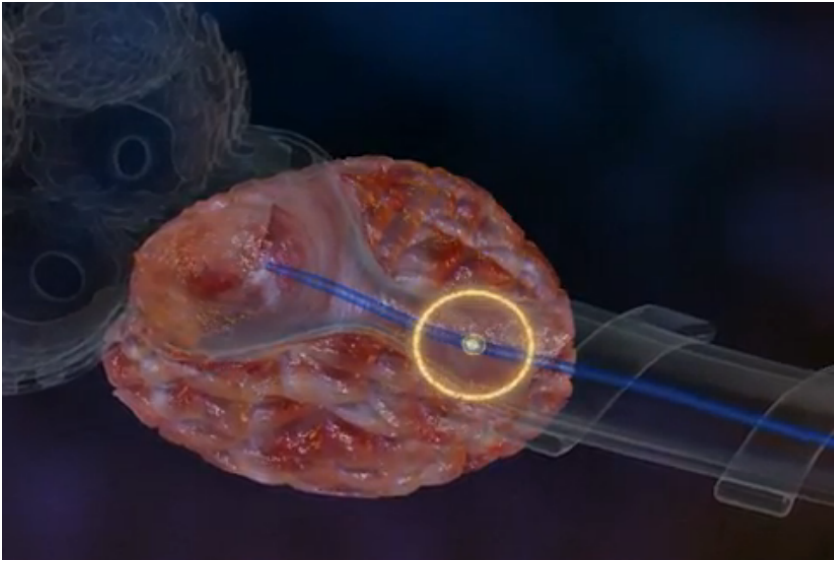


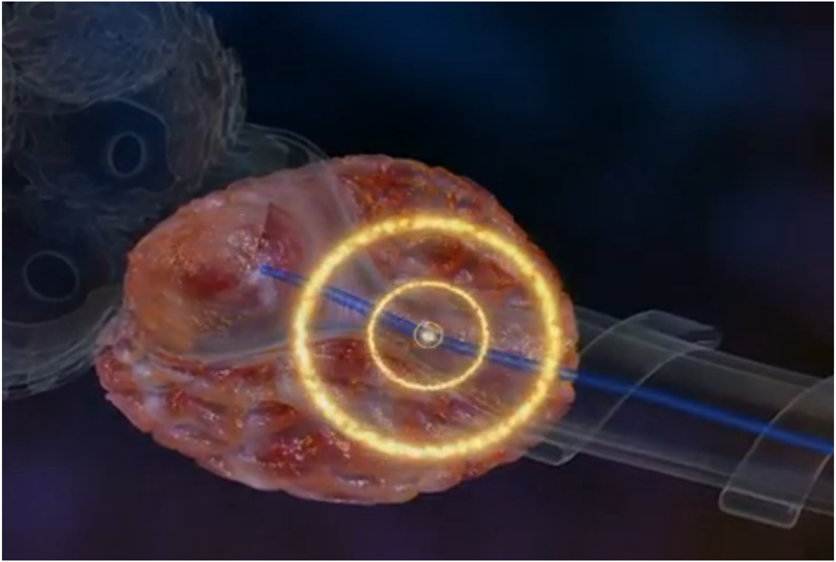
Internal beam therapy



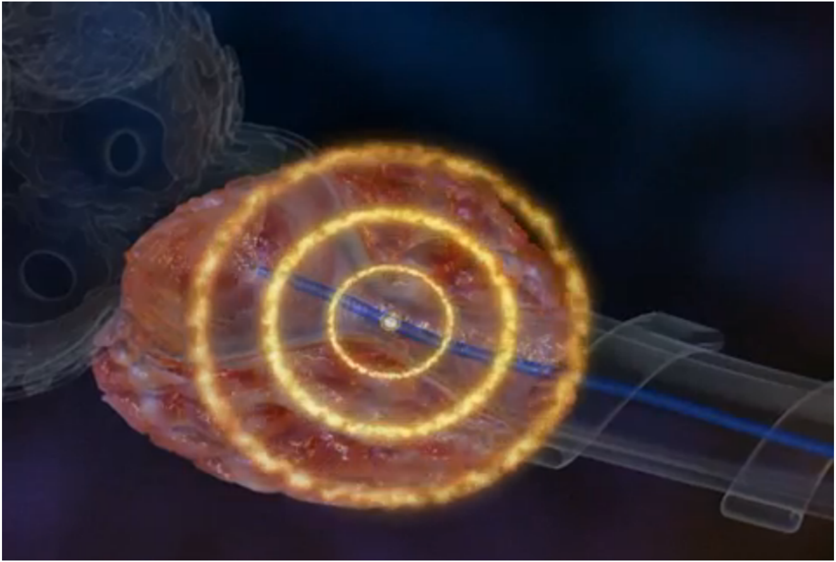
Internal beam therapy



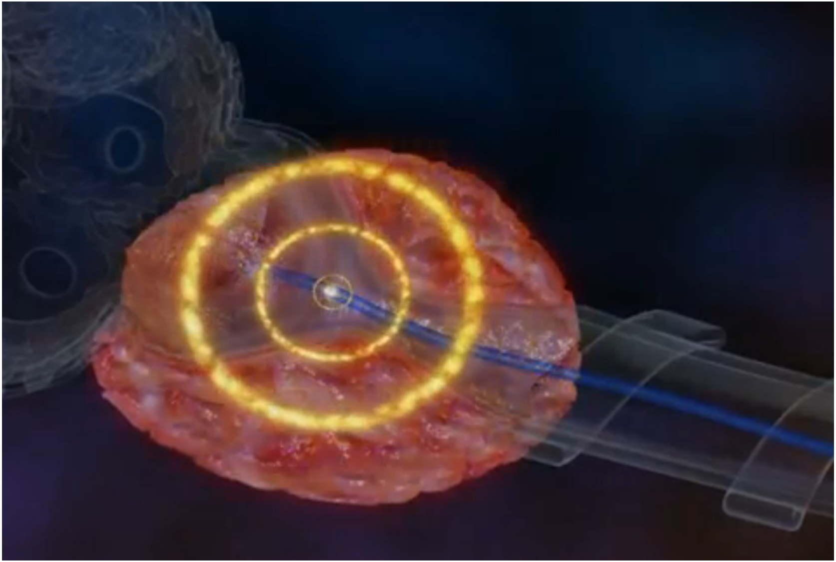
Internal beam therapy



Internal beam therapy



Internal beam therapy



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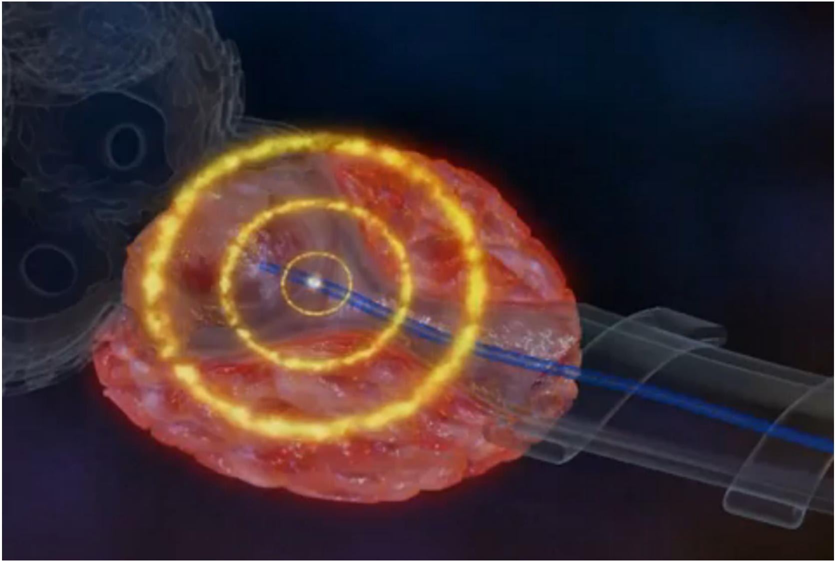

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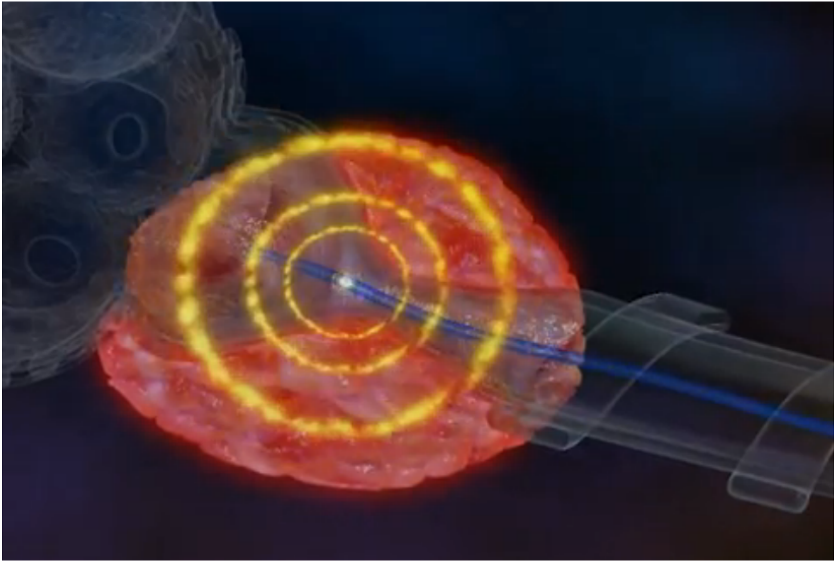
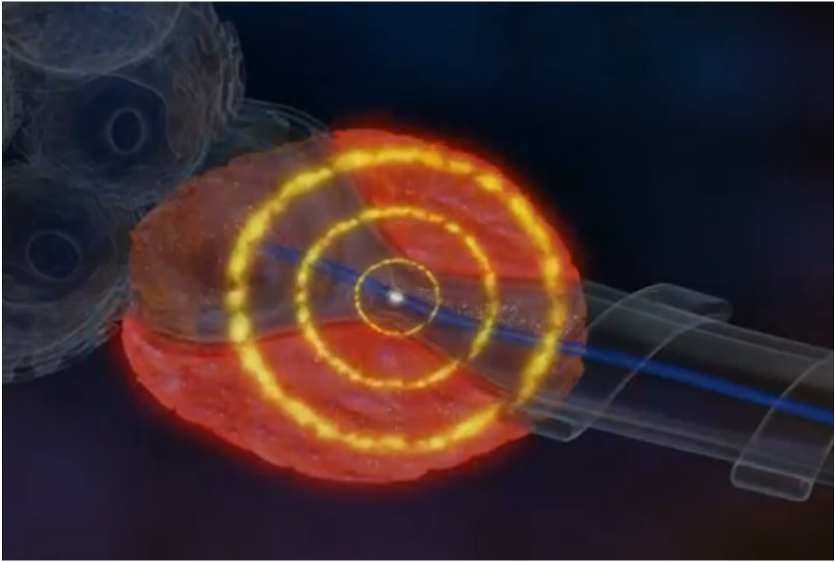


Internal beam therapy



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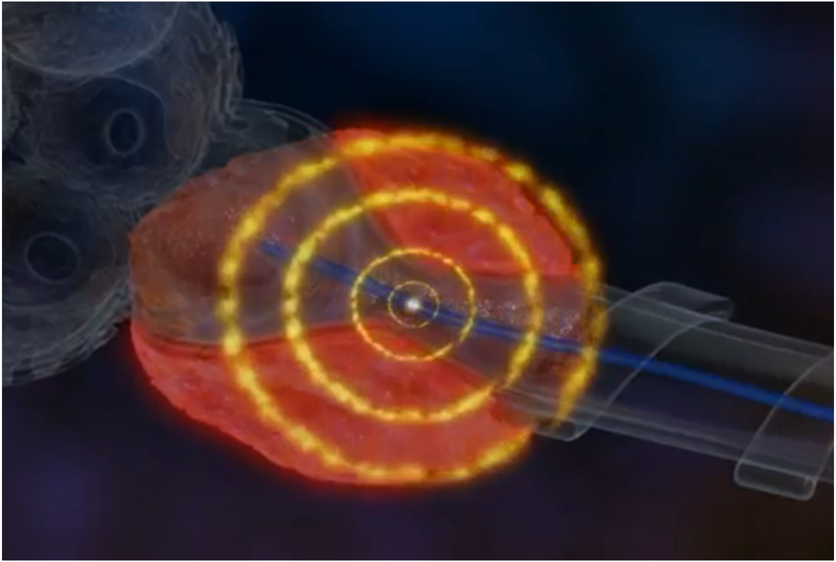

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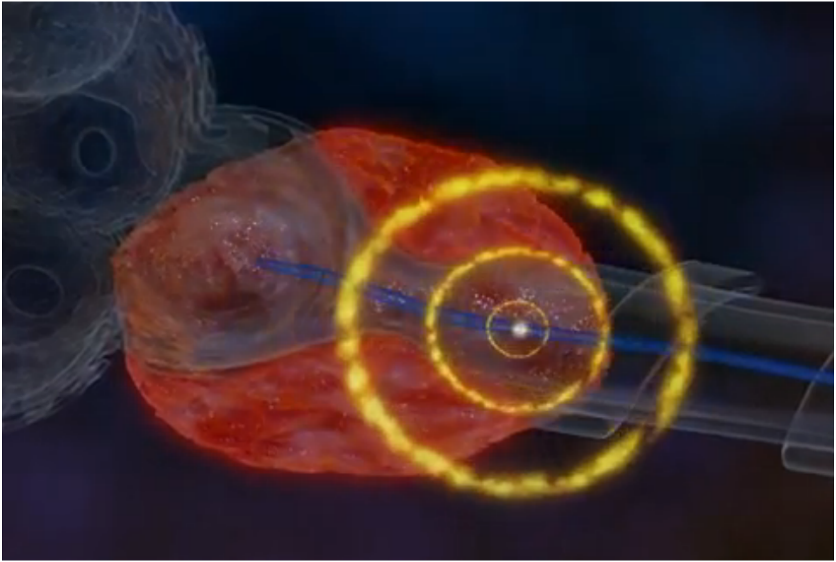

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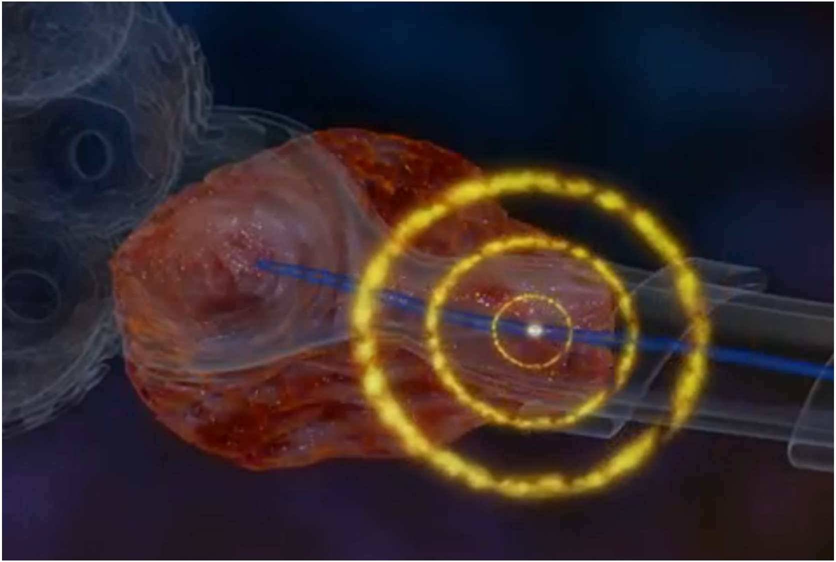

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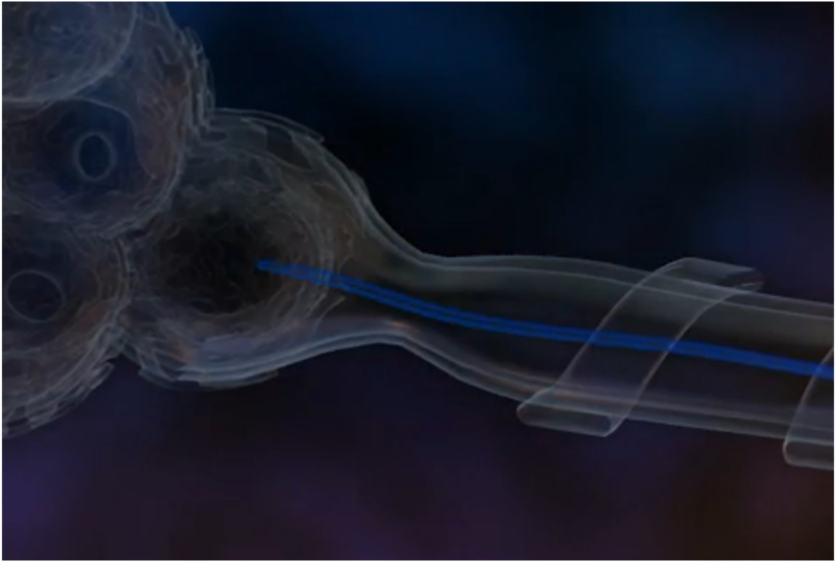
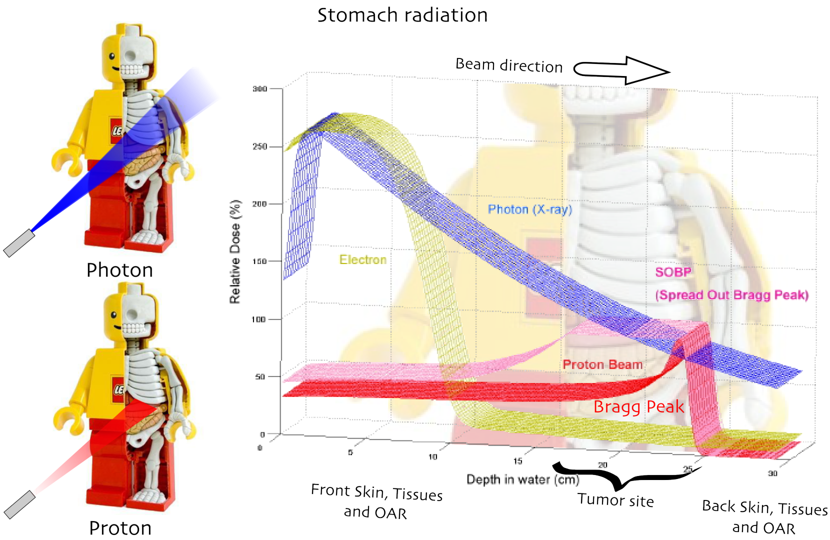


Different particles



Pros and Cons

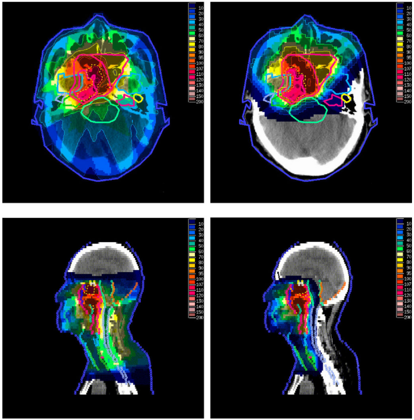


Figure 1: Taheri-Kadkhoda et al. Radiation Oncology 2008

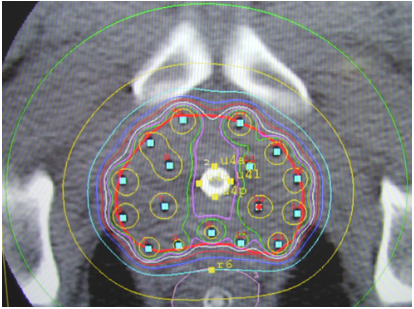


Figure 2: UCLA Brachytherapy Program

Protons and brachy therapies spare more healthy tissues

Pros and Cons

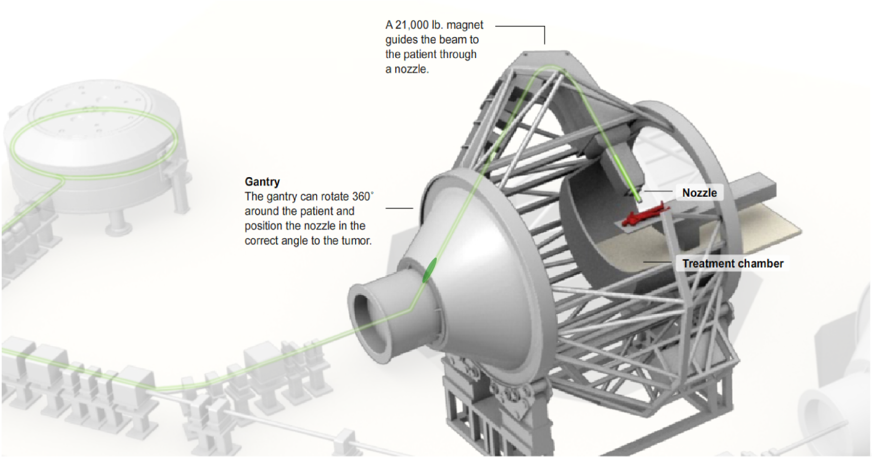


Figure 1: Protons center are expensive - 95 Millions euros, size of a building

Pros and Cons

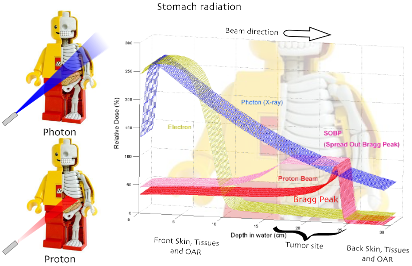


Figure 1: Bragg peak

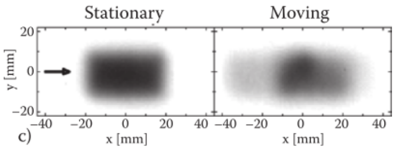


Figure 2: Motion sensitivity

Pros and Cons

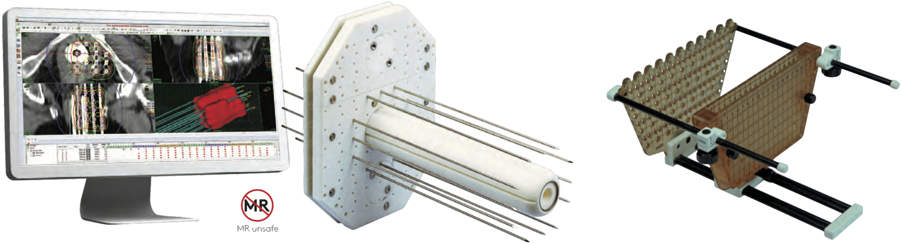


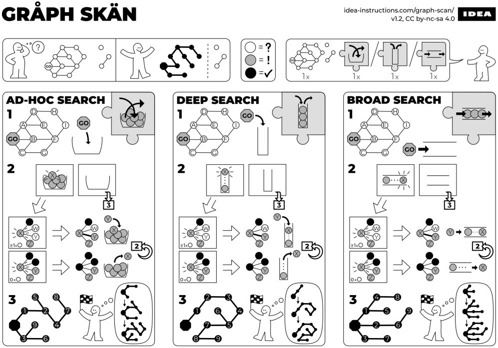
Figure 1: Brachy is invasive and needs catheter or needles to reach the tumor site

Main common problem

- Take into account specificities between patients or along the treatment for a single one due to variance arising in
  - Patient setup
  - Patient breathing / coughing
  - Patient heart-beat
  - Patient discomfort
  - Patient weight fluctuation
  - Patient implants
  - ...

What about the algorithmic in all this ?

- Binary matrices
- Stringology
- Pathways in graph
- Big data
- Deep learning



Multileaf collimators

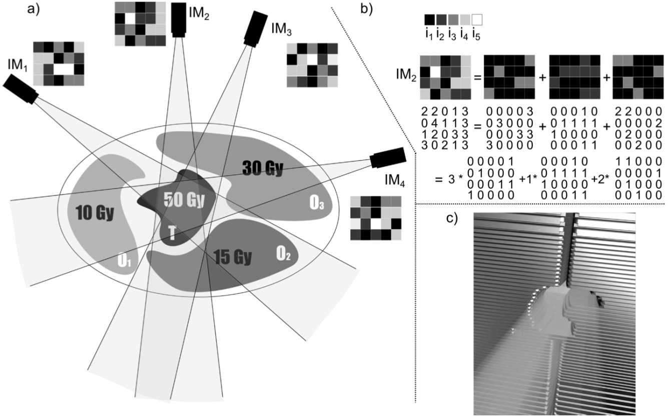


Fig. 1. *a) IMRT with some intensity matrices – shown in grayscale coded grids with 5 intensities (the lighter the color the higher the radiation intensity). b) A realization of* IM_2 *with* $i_1 = 0$, $i_2 = 1$, $i_3 = 2$, $i_4 = 3$, $i_5 = 4$. *c) MLC illustration from Varian*

- Optimize total and/or setup time

Multileaf collimators

- Minimizing the total beam-on time is solvable in linear time
- Minimizing the total setup time is Strongly NP-hard even for matrices with a single row
- We investigated algorithmic aspects of two technological variants in Sofsem 2014

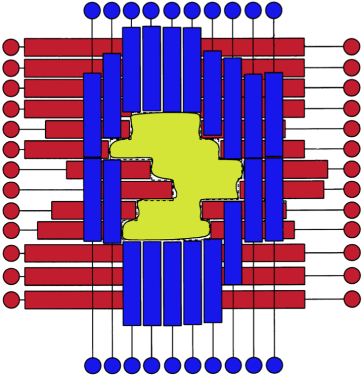
Multileaf collimators variants

$$\begin{bmatrix}1&4&2&5\\1&3&3&2\\1&3&5&5\\6&4&6&0\end{bmatrix} = \begin{bmatrix}0&0&0&1\\0&0&1&1\\0&0&1&1\\1&1&1&0\end{bmatrix} + \begin{bmatrix}0&0&0&1\\0&1&1&1\\0&0&1&1\\1&1&1&0\end{bmatrix} + \begin{bmatrix}0&1&1&1\\1&1&1&0\\0&1&1&1\\1&1&1&0\end{bmatrix} + \begin{bmatrix}1&1&1&1\\0&0&0&0\\1&1&1&1\\1&1&1&0\end{bmatrix}$$
$$+ \circlearrowleft + \begin{bmatrix}0&1&0&1\\0&0&0&0\\0&0&0&0\\1&0&1&0\end{bmatrix} + \begin{bmatrix}0&1&0&0\\0&1&0&0\\0&1&0&1\\1&0&1&0\end{bmatrix}$$

Figure 2: Rotating Collimator

- The Rotating MLC Decomposition problem is NP-Hard when minimizing either the total setup time or the total beam-on time
- Approximable with an additional overcost relative to size

Multileaf collimators variants



$$
\begin{bmatrix}
1 & 0 & 1 & 0 & 1 & \dots & 0 \\
0 & 1 & 0 & 1 & 0 & \dots & 1 \\
1 & 0 & 1 & 0 & 1 & \dots & 0 \\
0 & 1 & 0 & 1 & 0 & \dots & 1 \\
1 & 0 & 1 & 0 & 1 & \dots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 1 & 0 & 1 & 0 & \dots & 1
\end{bmatrix}
=
\begin{bmatrix}
1 & \uparrow & 1 & \uparrow & 1 & \dots & \uparrow \\
\leftarrow & + & \leftarrow & + & \leftarrow & \dots & + \\
1 & \uparrow & 1 & \uparrow & 1 & \dots & \uparrow \\
\leftarrow & + & \leftarrow & + & \leftarrow & \dots & + \\
1 & \uparrow & 1 & \uparrow & 1 & \dots & \uparrow \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
\leftarrow & + & \leftarrow & + & \leftarrow & \dots & +
\end{bmatrix}
+
\begin{bmatrix}
+ & \rightarrow & + & \rightarrow & + & \dots & \rightarrow \\
\uparrow & 1 & \uparrow & 1 & \uparrow & \dots & 1 \\
+ & \rightarrow & + & \rightarrow & + & \dots & \rightarrow \\
\uparrow & 1 & \uparrow & 1 & \uparrow & \dots & 1 \\
+ & \rightarrow & + & \rightarrow & + & \dots & \rightarrow \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
\uparrow & 1 & \uparrow & 1 & \uparrow & \dots & 1
\end{bmatrix}
$$

Figure 2: Multi-Layer Multileaf Collimator

- The Dual-MLC Decomposition problem is NP-Hard when minimizing the total setup time.

Modulated brachytherapy

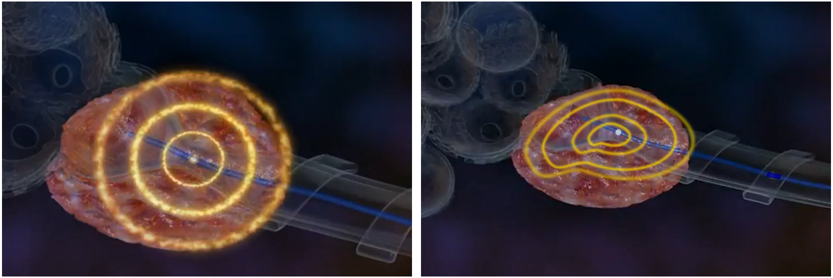
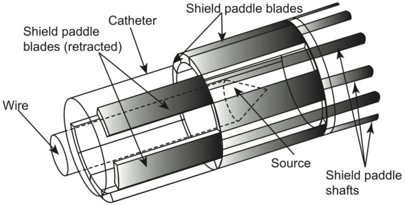
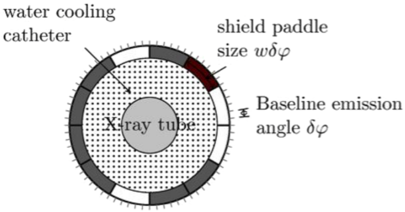


Figure 3: Modulated brachytherapy

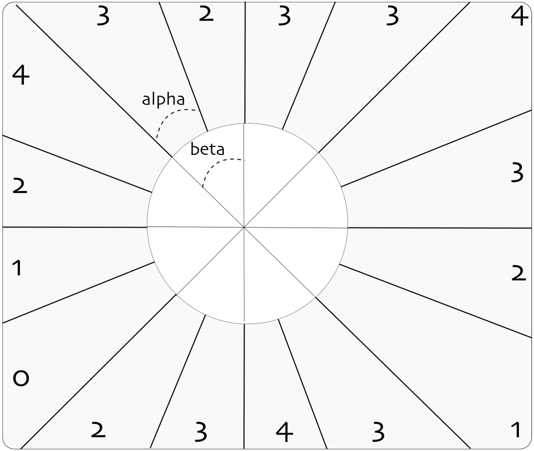
- Conformation to the shape of the tumor site
- In practice, computation are done relatively to dose absorption in water

Tunable shield



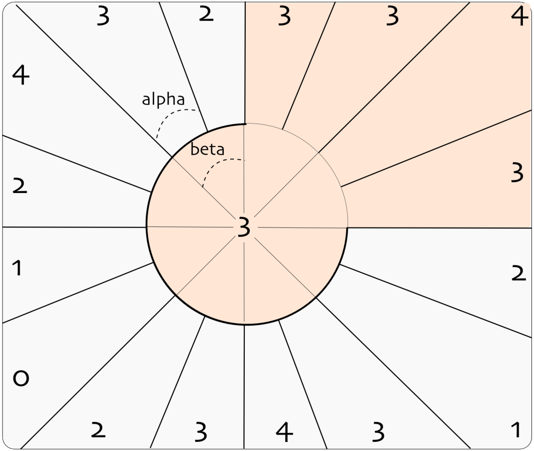


Tunable shield



- Circular integer word decomposition into circular binary words under constraints

Tunable shield



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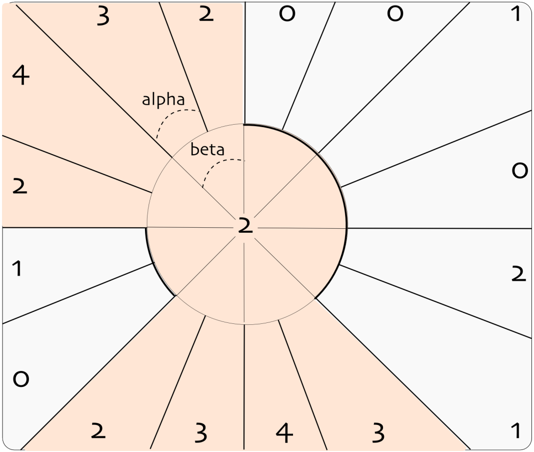

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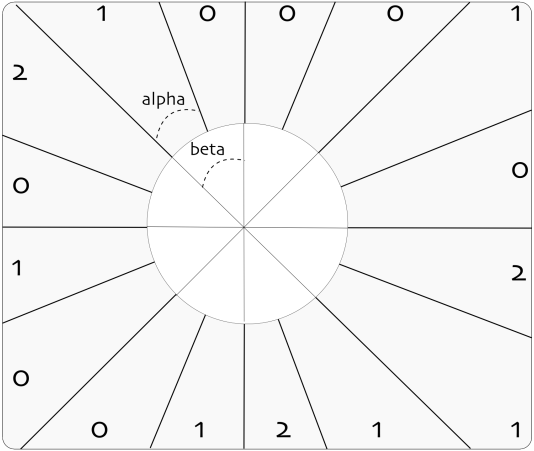

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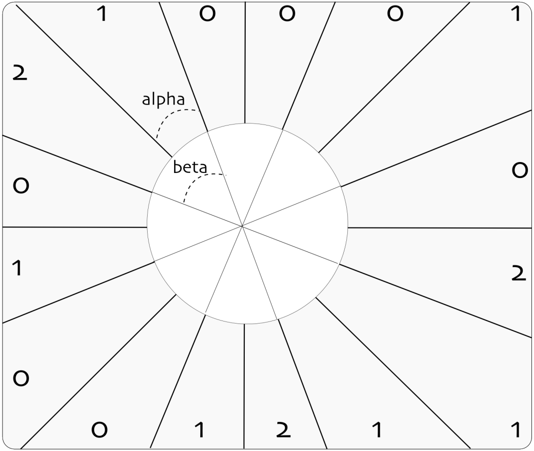

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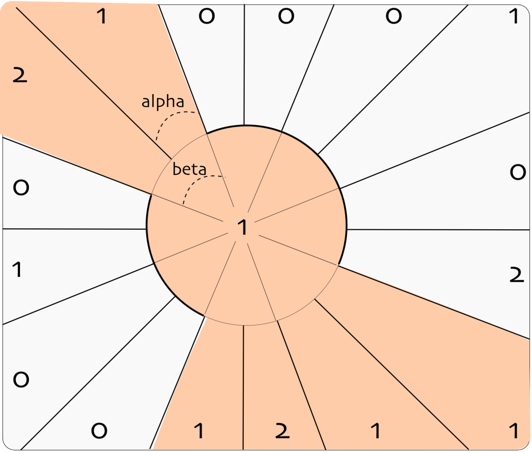
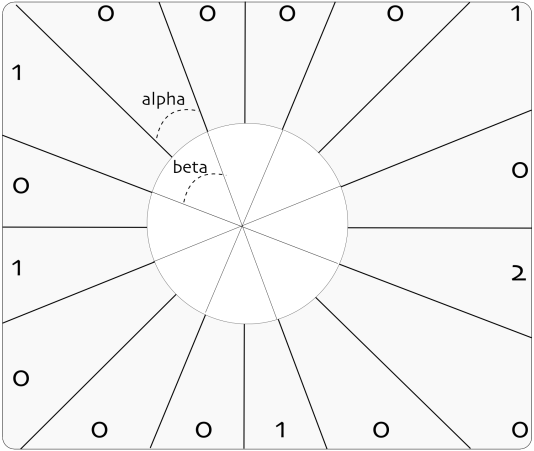


- Circular integer word decomposition into circular binary words under constraints

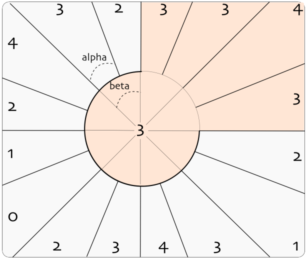
Tunable shield



- Circular integer word decomposition into circular binary words under constraints

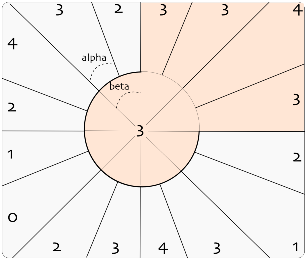
Tunable shield

- The shield configuration can be considered as fixed or dynamic
- Provided with or without rotation capabilities
- Allowing or not irradiation overdoses
- We investigated algorithmic aspects of those variants in IWOCA 2016



Known results

- Provided with one single fixed configuration
- Allowing overdose, it can be solved in O(N log N).
- Forbidding overdose, it can be solved in O(N)



Known results

- Considering multiple shield configurations allowed
- Achieving the optimal difference between the prescribed dose and the actual total delivered dose using a minimal number of shield configurations
- Given an upper bound on the number of shield configurations, achieving the minimum reachable difference
- Both are NP-hard even when each shield sector is associated to a even number of consecutive patient volumes
- But can be approximated in polynomial time within a factor of log of the max prescribed dose of the optimum

Customized cylindrical shields for brachytherapy

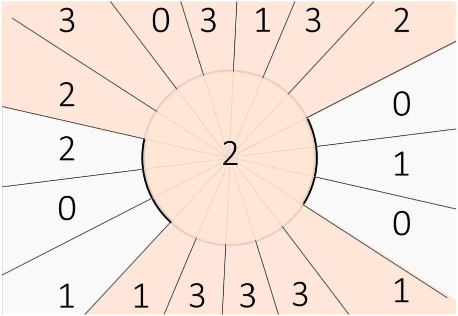
- Manufacturing a given single best shield for a given patient (3D Metal printing)
- Assume that the physical precision of our process is limited (lower bounds on the size of a closed or open sector of a produced shield)
- We investigated algorithmic aspects of the corresponding problem in CPM 2018

Customized cylindrical shields for brachytherapy

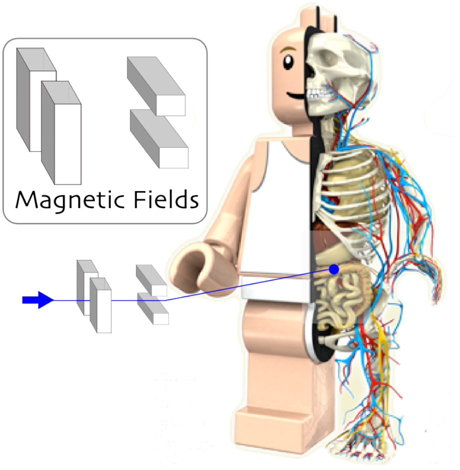
- Given a circular integer word w, the cylindrical shield to be designed can be seen as a constrained circular binary word of the same length where, when we replace each 1 by the selected irradiation time t, the Manhattan distance to w is minimal.
- Constraints on the circular binary word are according to the minimal length for an opening, and for a closed sector between two openings

Customized cylindrical shields for brachytherapy

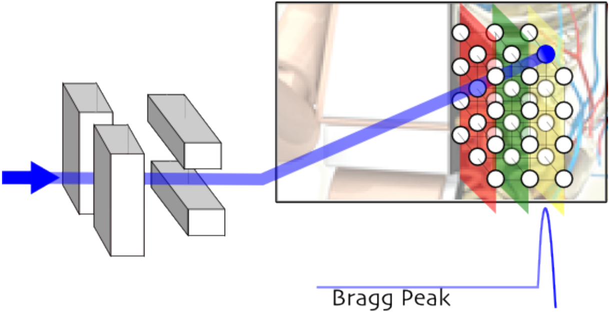
- A pseudo-polynomial time algorithm of complexity $O(|w| * tmax * l^3)$ exists with t_{max} the maximal time of an irradiation and l the maximum sector size
- $w = 013331102230313210$ with $l_0 = 3,\ l_1 = 5$



Pencil Beam Discrete Scanning

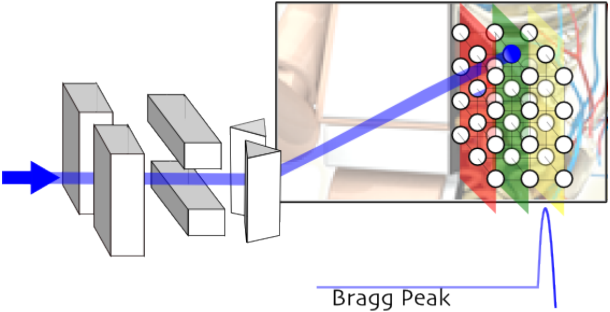


Pencil Beam Discrete Scanning



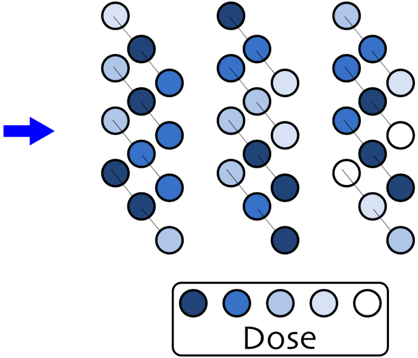
The beam is turned off between the spot positions

Pencil Beam Discrete Scanning



The beam is turned off between the spot positions

Treatment Planning



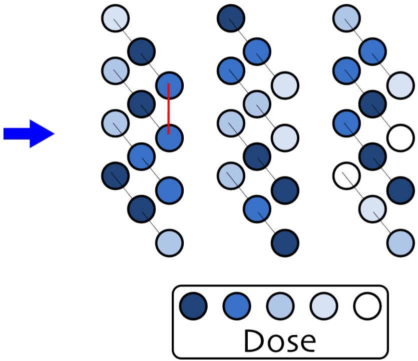
Plan:

...

Step k: Energy, x coordinate , y coordinate, duration

...

Treatment Planning



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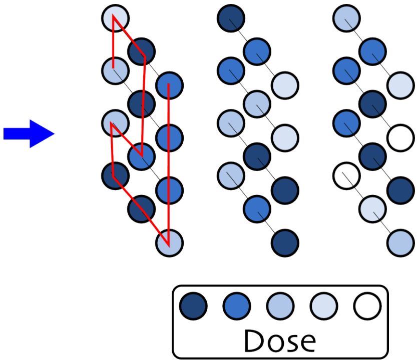
Treatment Planning



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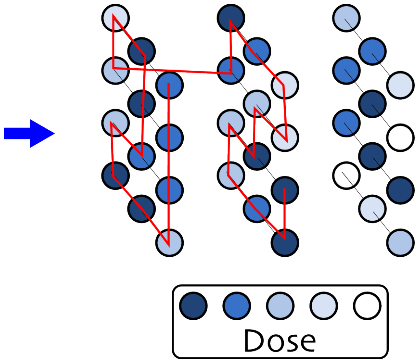
Treatment Planning



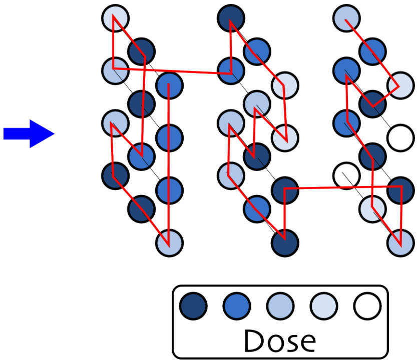
Plan:

...

Step k: Energy, x coordinate , y coordinate, duration

...

Treatment Planning



Plan:

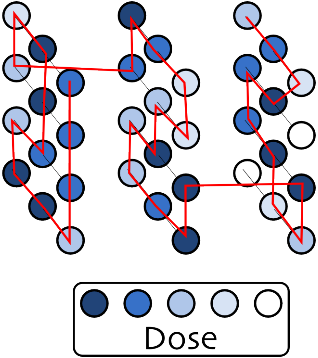
...

Step k: Energy, x coordinate , y coordinate, duration

...

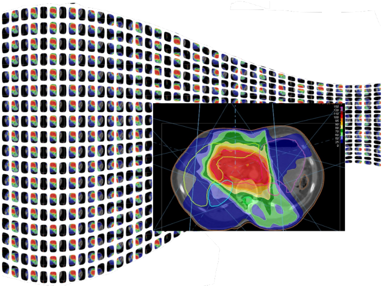
Optimization of paths

- Not so much investigated from the algorithmic point of view
- Necessity to take into account motion sensitivity
- We proposed an "An open-source motion simulator for proton therapy algorithmic aspects" (MSPT) in the PhD thesis of Paul Morel 2015 which reflects the consequences of motion on a treatment



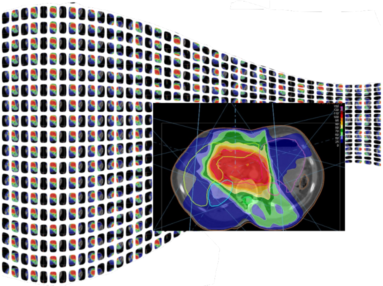
What about big data and deep learning ?

- How to provide personal treatment plans in real time like fashion ?
- Take advantage from past treatment plans
  - Gathering treatment plans
  - Storing them in an efficient way
  - Query them in real time



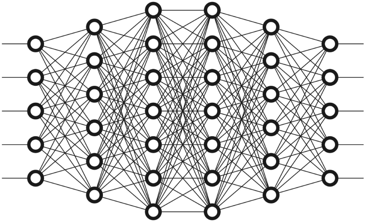
What about big data and deep learning ?

- How to provide personal treatment plans in real time like fashion ?
- Take advantage from past treatment plans
  - Avoid redundant computation
  - Being able to start from a realistic draft of the treatment plan rather than from scratch
  - Compute alternative plans to react in real time



Learning plans rather than computed them

- Underline physics is complicated
- Rather than trying to compute it faster, could one learn it somehow ?



Conclusion

- Radiation therapies provides lots of interesting problems
- One can easily find its own algorithmic playground