Communicating Finite-State Machines, First-Order Logic, and Star-Free Propositional Dynamic Logic

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CAALM
Chennai Mathematical Institute
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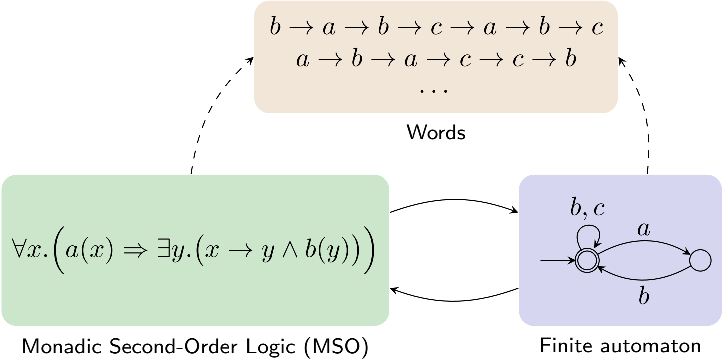
Slides courtesy of Marie Fortin.

Büchi-Elgot-Trakhtenbrot Theorem ('60s)



$\mathrm{MSO}[\to] = \text{Finite Automata}$

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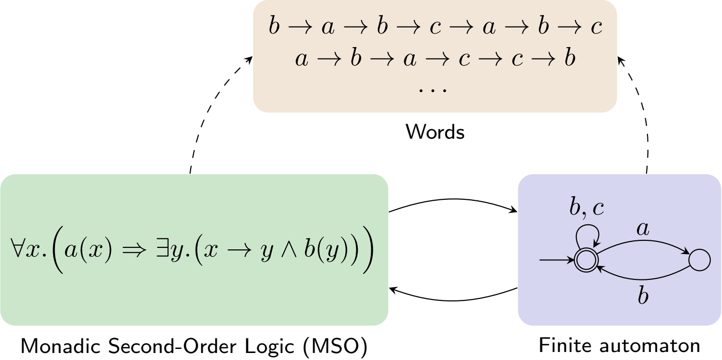

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$\exists X_0 \dots \exists X_n . \varphi$ with $\varphi \in \mathsf{FO}[\to]$

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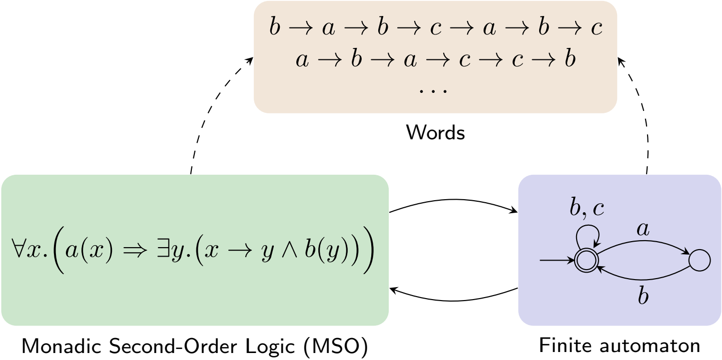

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Extended to trees [Thatcher-Wright '68], Mazurkiewicz traces [Thomas '90], nested words [Alur-Madhusudan '04], data words [Bojańczyk et al. '06], weighted automata [Droste-Gastin '05], ...

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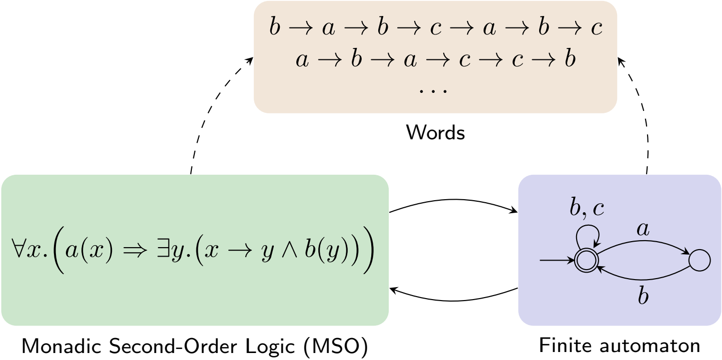

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Goal: A Büchi-like theorem for message-passing systems

Büchi-Elgot-Trakhtenbrot Theorem ('60s)

$b \to a \to b \to c \to a \to b \to c$

$a \to b \to a \to c \to c \to b$

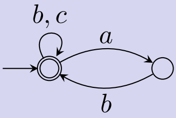
$\ldots$

Words

$$\forall x.\Big(a(x) \Rightarrow \exists y.\big(x \to y \wedge b(y)\big)\Big)$$

Monadic Second-Order Logic (MSO)

Synthesis



Finite automaton

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$$\boldsymbol{q} \qquad a \longrightarrow a \longrightarrow a \longrightarrow a \longrightarrow a \longrightarrow a \longrightarrow a \longrightarrow a \longrightarrow a \longrightarrow a$$

$$\boldsymbol{r} \qquad a \longrightarrow b \longrightarrow b \longrightarrow a \longrightarrow a \longrightarrow c \longrightarrow a \longrightarrow a \longrightarrow a$$

The model

- Fix finite set of processes and finite alphabet (e.g., $P = \{p, q, r\}$ and $\Sigma = \{a, b, c\}$)
- Reliable unbounded point-to-point FIFO channels

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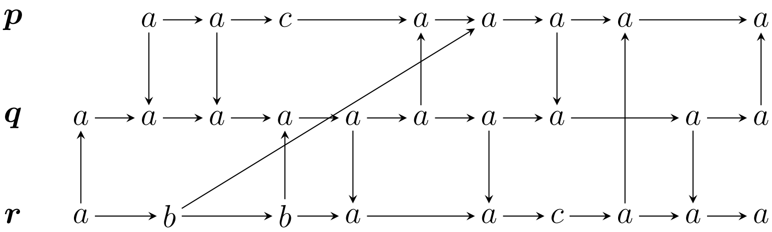

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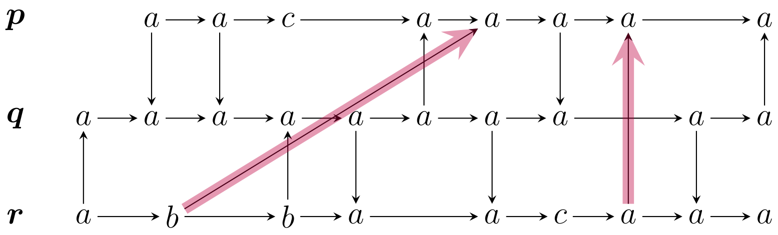

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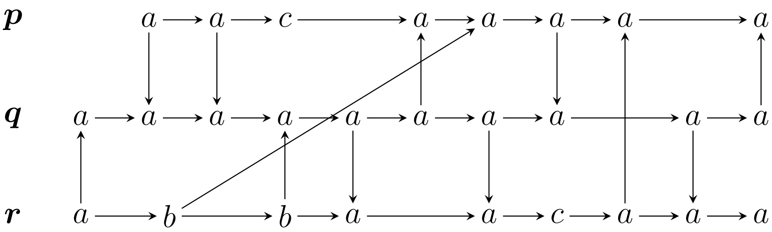
- Partial order semantics: Message Sequence Charts (MSC)



Communicating finite-state machines (CFMs)

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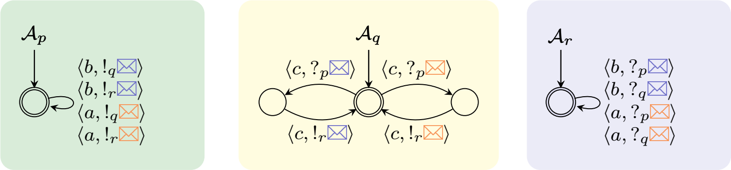

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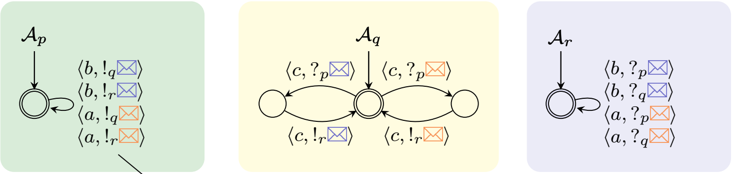

"send $\boxtimes$ to process r"

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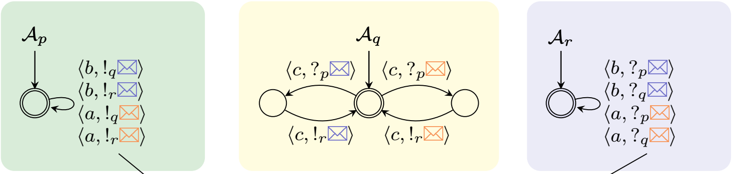

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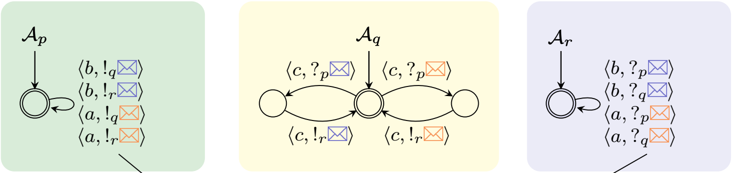

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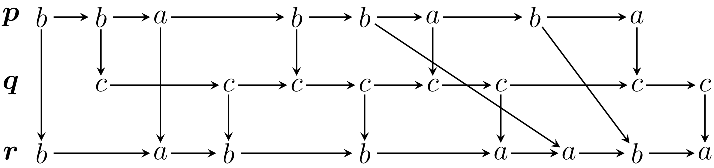
Run of CFMs on MSCs:



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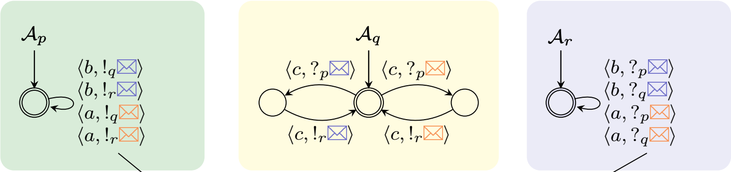

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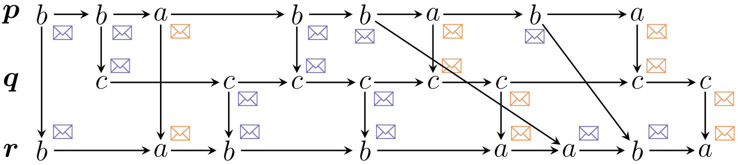

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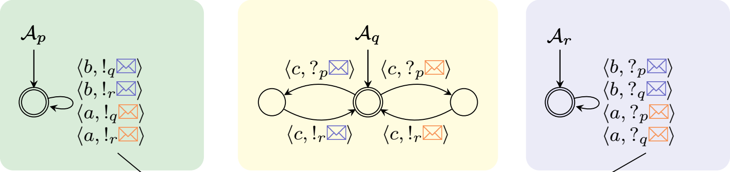

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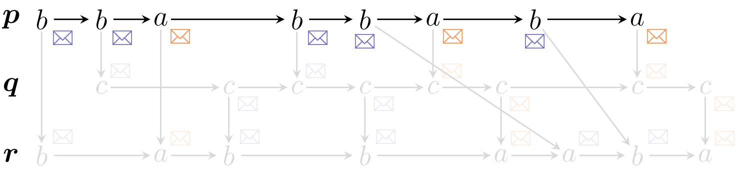

$\in L(\mathcal{A}_p)$

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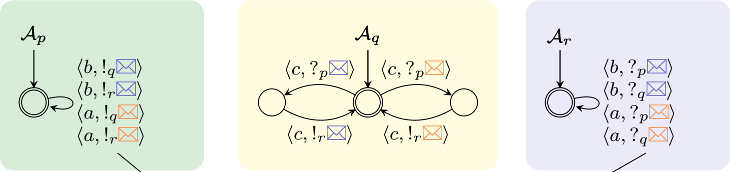

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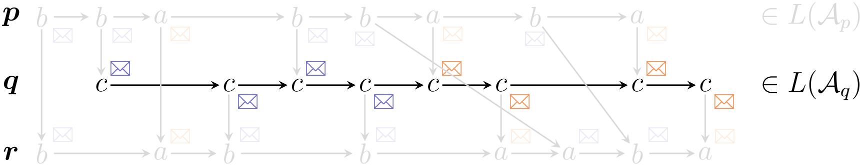

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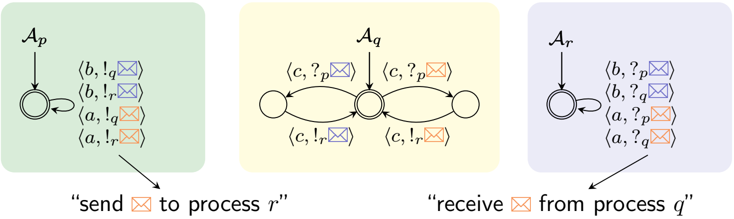

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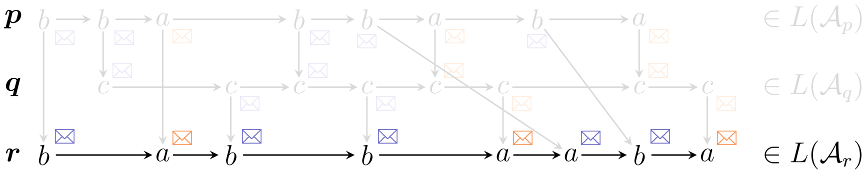

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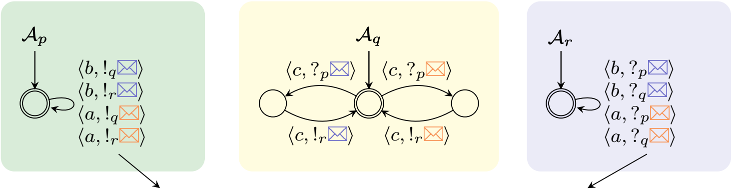

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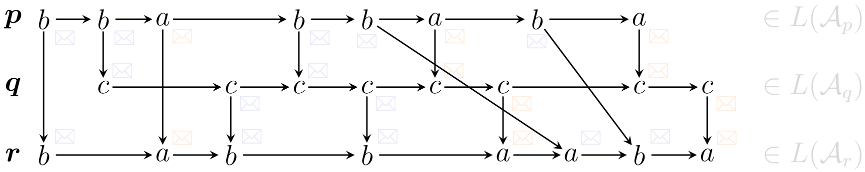

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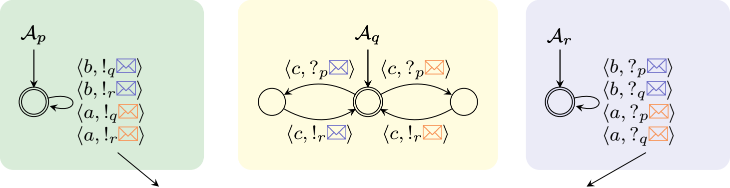


"send $\boxtimes$ to process r" $\qquad$ "receive $\boxtimes$ from process q"

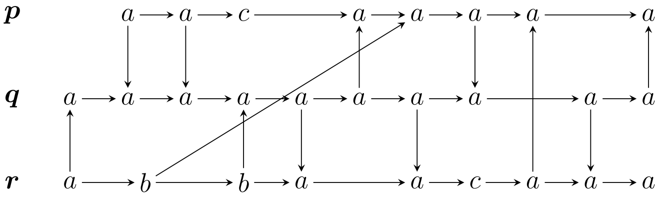
Remarks

- The emptiness problem for CFMs is **undecidable**.
- CFMs are inherently **non-deterministic**.

[Genest-Kuske-Muscholl '07]

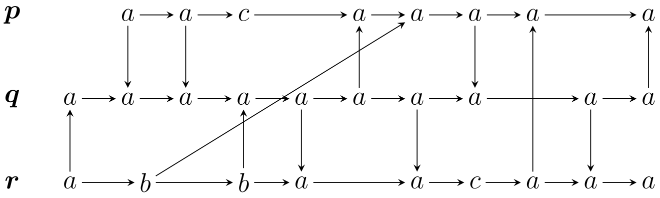
Monadic Second-Order Logic (MSO) over MSCs

$\varphi ::=$



Monadic Second-Order Logic (MSO) over MSCs

$\varphi ::= \quad a(x) \mid p(x)$ label/process of event x



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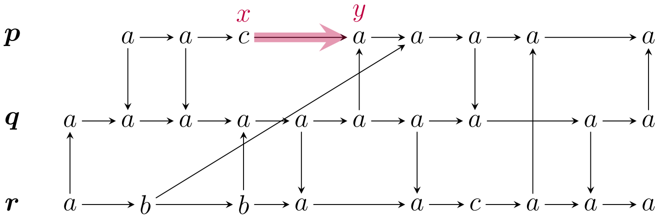

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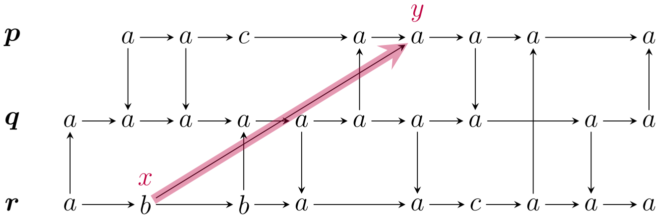

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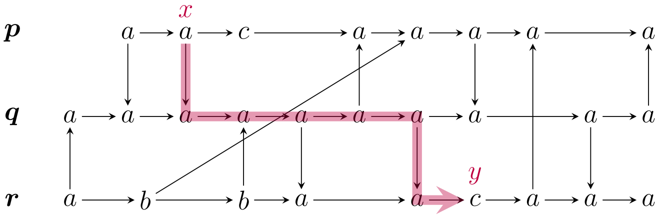

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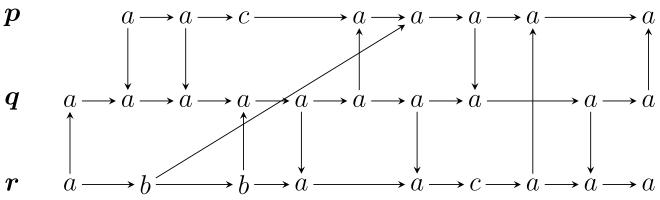
	$\mid \neg\varphi \mid \varphi \vee \varphi \mid \exists x.\varphi \mid \exists X.\varphi \mid x \in X$	



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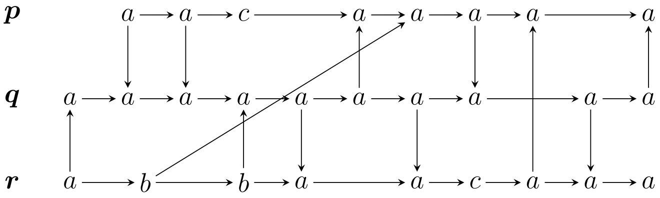

Mutual exclusion: $\neg(\exists x.\exists y.c(x) \wedge c(y) \wedge \; x \parallel y \;)$

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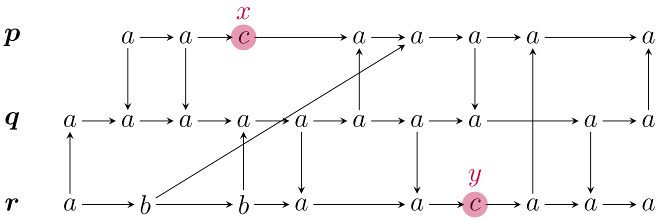

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$\neg(x \leq y) \wedge \neg(y \leq x)$

Büchi-like theorems for CFMs

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When channels are **bounded**:

Büchi-like theorems for CFMs

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Theorem (Henriksen-Mukund-Narayan Kumar-Sohoni-Thiagarajan '05)

Over universally bounded MSCs, CFM $=$ MSO$[\searrow, \rightarrow, \leq]$.

Theorem (Genest-Kuske-Muscholl '06)

Over existentially bounded MSCs, CFM $=$ MSO$[\searrow, \rightarrow, \leq]$.

General case

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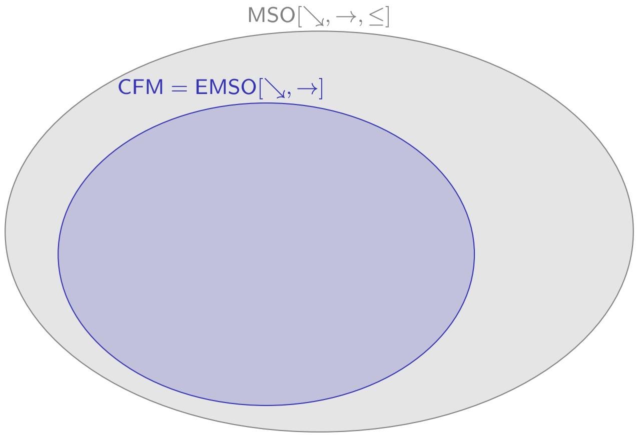

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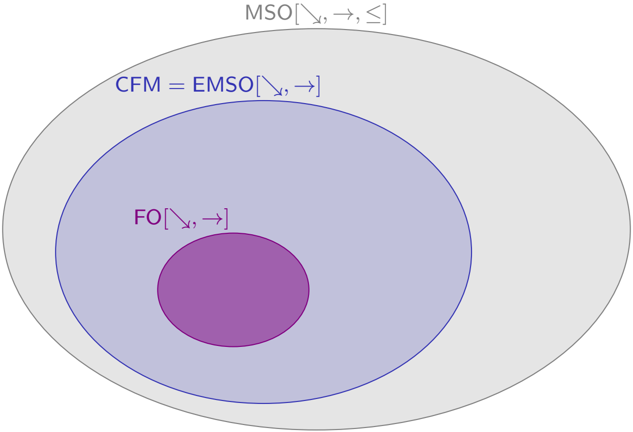

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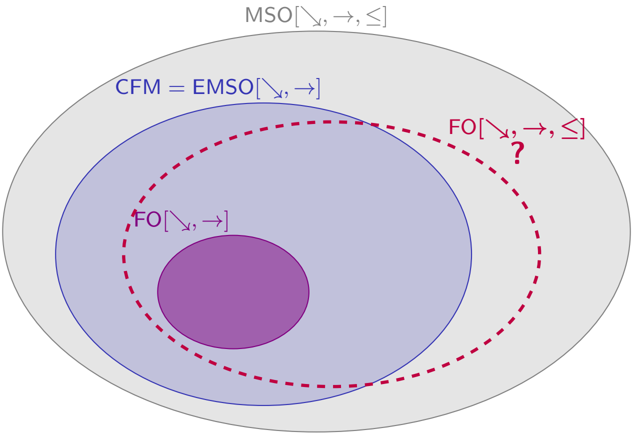

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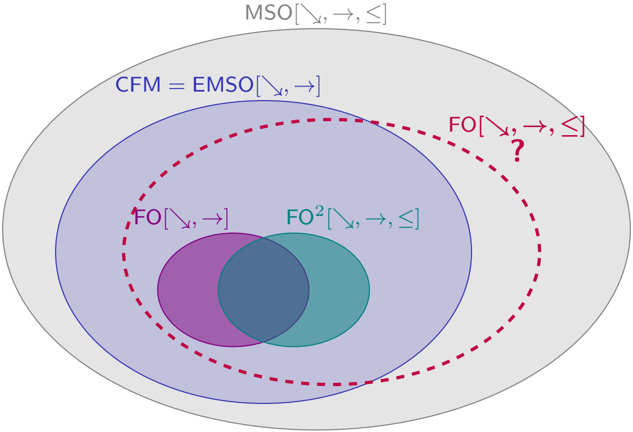

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[B.-Fortin-Gastin '18] $\mathrm{CFM} = \mathrm{EMSO}^2[\searrow, \rightarrow, \leq]$

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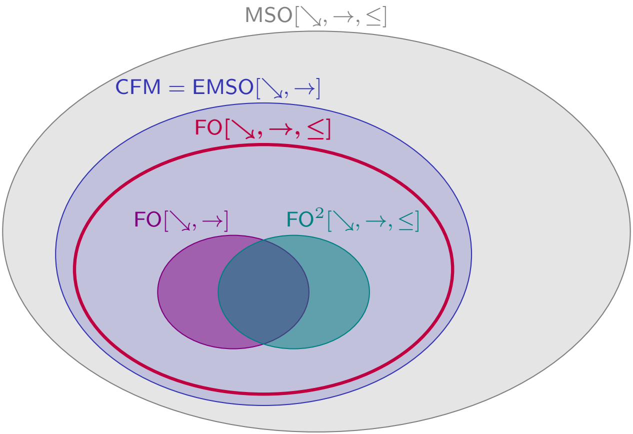

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Main result: $\mathsf{CFM} = \mathsf{EMSO}[\searrow, \rightarrow, \leq]$

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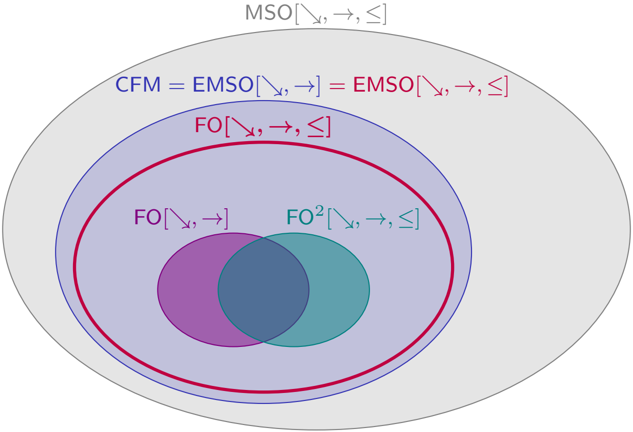

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Translation from FO to CFMs

Goal

$FO[\searrow, \rightarrow, \leq]$ sentence φ → CFM $\mathcal{A}$ such that $L(\mathcal{A}) = L(\varphi)$

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FO$[\searrow, \rightarrow, \leq]$ sentence φ $\longrightarrow$ CFM $\mathcal{A}$ such that $L(\mathcal{A}) = L(\varphi)$

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- Techniques used for previous cases do not apply here

Translation from FO to CFMs

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FO[$\searrow, \rightarrow, \leq$] sentence φ → PDL$_{sf}$ formula $\widetilde{\varphi}$ → CFM $\mathcal{A}$ such that $L(\mathcal{A}) = L(\varphi)$

- CFMs are not closed under complementation – no direct induction on φ
- Techniques used for previous cases do not apply here

Solution: go through an intermediate language: "Star-free" Propositional Dynamic Logic (with Loop and Converse)

A simple modal logic for MSCs: $\mathsf{PDL}_{\mathsf{sf}}^{-}$

$\varphi, \psi ::=$

A simple modal logic for MSCs: PDL_{sf}^{-}

$$\varphi, \psi ::= \quad a \mid p \mid \varphi \vee \varphi \mid \neg\varphi$$

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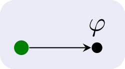
$$\mid \langle \rightarrow \rangle\, \varphi$$



("Next φ")

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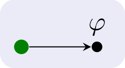

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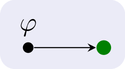

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A simple modal logic for MSCs: $\mathrm{PDL}_{\mathrm{sf}}^{-}$

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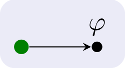

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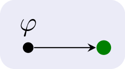

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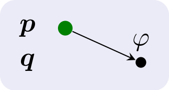

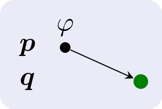

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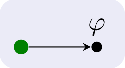

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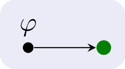

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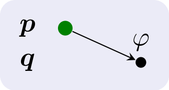

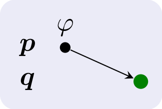

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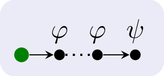

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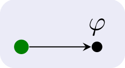

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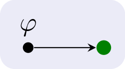

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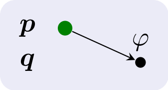

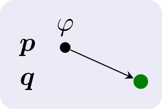

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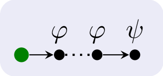

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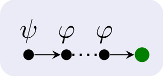

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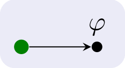

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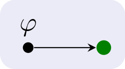

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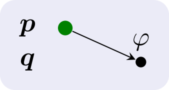

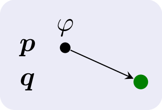

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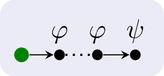

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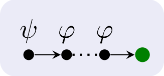
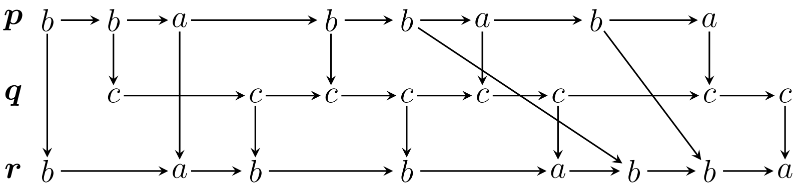


(“φ Since ψ”)

$\mid \langle \mathrm{jump}_{p,q} \rangle\, \varphi$

Examples

$P = \{p, q, r\}$, $\Sigma = \{a, b, c\}$



"On q, a receive from p is directly followed by a send to r":

$$\boxed{\langle {}^{p}\!\nwarrow_{q} \rangle\ \mathit{true}} \implies \langle \rightarrow \rangle\ \boxed{\langle {}^{q}\!\searrow_{r} \rangle\ \mathit{true}}$$

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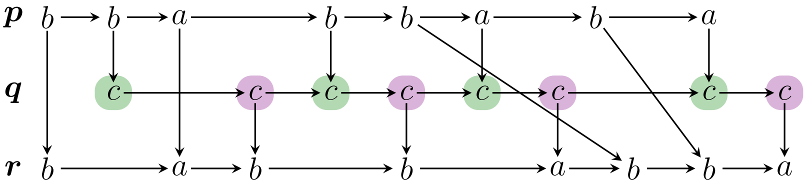

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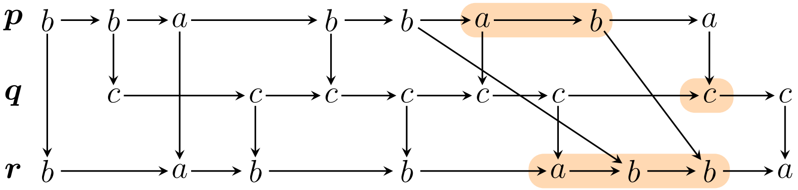

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"All events strictly in between the current and the last event on a process are bs":

$$\langle \xrightarrow{b} \rangle\ \neg \langle \rightarrow \rangle\ \mathit{true}$$

Expressivity of $\mathsf{PDL}^{-}_{\mathsf{sf}}$

Over words ($|P| = 1$):

$$\begin{tikzcd}
\text{FO} \arrow[r] \arrow[d, bend left] & \text{Finite Automata} \\
\text{LTL} \arrow[u, bend left] \arrow[ur, bend right] &
\end{tikzcd}$$

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Over general MSCs:

$$\begin{tikzcd}
\mathsf{FO} & \mathsf{CFMs} \\
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\end{tikzcd}$$

Expressivity of $\mathrm{PDL}_{\mathrm{sf}}^{-}$

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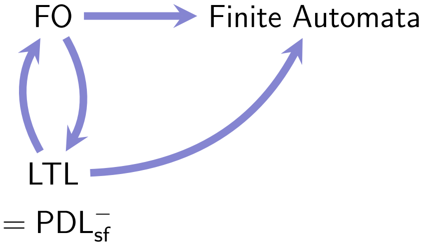

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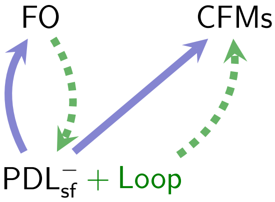

Star-free Propositional Dynamic Logic ($\mathrm{PDL}_{\mathrm{sf}}$)

[Fischer-Ladner 1979] (PDL)

Event formulas

$\varphi ::= a \mid p \mid \varphi \vee \varphi \mid \neg\varphi$

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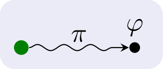

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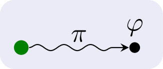

$$\pi ::= \rightarrow \mid \leftarrow \mid {}^{p}\!\searrow_{q} \mid {}^{p}\!\nwarrow_{q} \mid \mathsf{jump}_{p,q} \mid \xrightarrow{\varphi} \mid \xleftarrow{\varphi}$$

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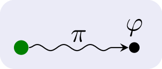

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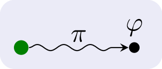

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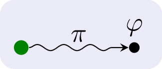

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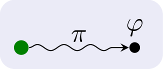
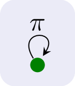


$$\mid \mathsf{Loop}(\pi)$$



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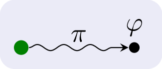

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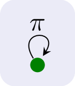

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$$\mid \pi\cup\pi \mid \pi\cap\pi \mid \pi^{\mathsf{c}}$$

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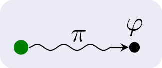

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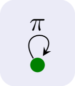

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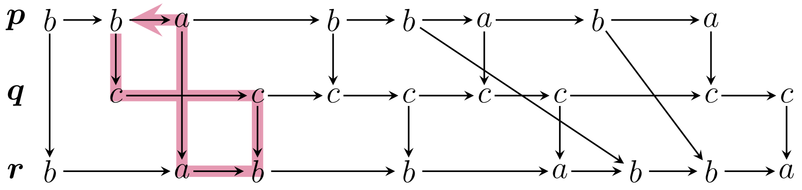

$$\varphi = \mathsf{Loop}({}^{p}\!\searrow_{q} \cdot \rightarrow \cdot {}^{q}\!\searrow_{r} \cdot \{b\}? \cdot \leftarrow \cdot {}^{p}\!\nwarrow_{r} \cdot \leftarrow)$$

Main results

$$\boxed{\mathrm{FO}[\searrow, \rightarrow, \leq]} \xrightarrow{(1)} \boxed{\mathrm{PDL}_{\mathrm{sf}}} \xrightarrow{(2)} \boxed{\mathrm{CFMs} = \mathrm{EMSO}[\searrow, \rightarrow]}$$

Main results

$$
\begin{tikzcd}
\boxed{\mathsf{FO}[\searrow, \rightarrow, \leq]} \arrow[rr, "(1)"] & & \boxed{\mathsf{PDL}_{\mathsf{sf}}} \arrow[dl] \arrow[r, "(2)"] & \boxed{\mathsf{CFMs} = \mathsf{EMSO}[\searrow, \rightarrow]} \\
& \boxed{\mathsf{FO}^3[\searrow, \rightarrow, \leq]} \arrow[ul] & &
\end{tikzcd}
$$

From $\mathrm{PDL}_{\mathrm{sf}}$ to FO^3

- Any $\mathrm{PDL}_{\mathrm{sf}}$ event formula φ can be transformed into an FO^3 formula $\widetilde{\varphi}(x)$ with one free variable.

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$$p \quad \rightsquigarrow \quad p(x)$$

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$$\begin{array}{rcl} p & \rightsquigarrow & p(x) \\ \langle \pi \rangle \varphi & \rightsquigarrow & \big(\exists y. \widetilde{\varphi}(y) \wedge \widetilde{\pi}(x,y)\big)(x) \end{array}$$

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$$\mathsf{PDL}_{\mathsf{sf}} \subseteq \mathsf{FO}^3 \subseteq \mathsf{FO}$$

From PDL_{sf} to FO^3

- Any PDL_{sf} event formula φ can be transformed into an FO^3 formula $\widetilde{\varphi}(x)$ with one free variable.

$$\begin{array}{rcl} p & \rightsquigarrow & p(x) \\ \langle \pi \rangle \varphi & \rightsquigarrow & \big(\exists y. \widetilde{\varphi}(y) \wedge \widetilde{\pi}(x,y)\big)(x) \end{array}$$

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$$\text{PDL}_{\text{sf}} \subseteq \text{FO}^3 \subseteq \text{FO} \subseteq \text{PDL}_{\text{sf}}$$

From FO to $\mathsf{PDL}_{\mathsf{sf}}$

Theorem

Any FO formula $\Phi(x_1, \ldots, x_n)$ can be rewritten as

$$\Phi(x_1, \ldots, x_n) \equiv \bigvee \bigwedge \pi(x_i, x_j) \quad \text{where } \pi \in \mathsf{PDL}_{\mathsf{sf}}$$

Proof: By induction.

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Any FO formula $\Phi(x_1)$ can be rewritten as

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From FO to $\mathsf{PDL_{sf}}$

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Any FO formula $\Phi(x_1)$ can be rewritten as

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Proof: By induction.

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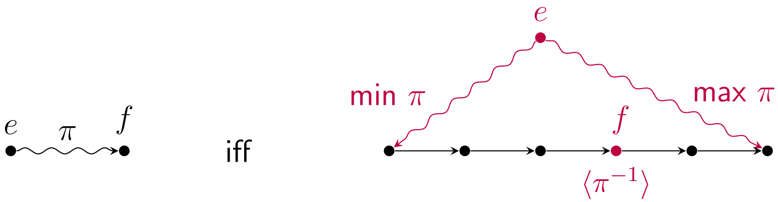
Proof: By induction. Two interesting cases:

- negation
- existential quantification

Key Lemma

For all $\pi \in \mathsf{PDL}_{\mathsf{sf}}$,



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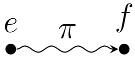
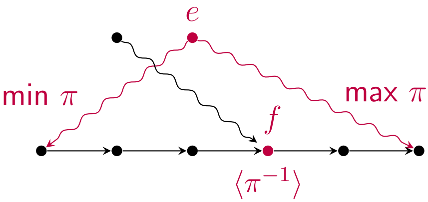


iff



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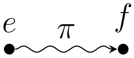

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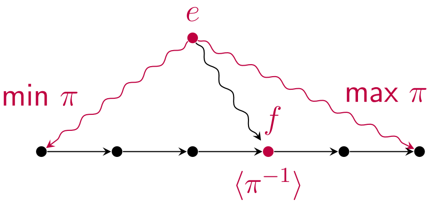

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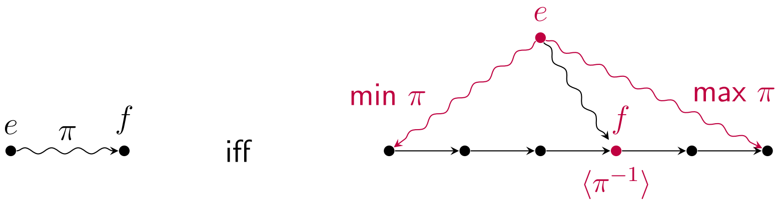

- Monotonicity of path formulas.
- Relies on FIFO behavior.
- Proof is by simple induction.

Negation

$$\neg \bigvee \bigwedge \pi(x_i, x_j) \equiv \bigwedge \bigvee \neg\pi(x_i, x_j)$$

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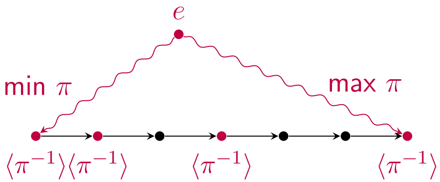

$\pi^{\mathsf{c}} \equiv$

Negation

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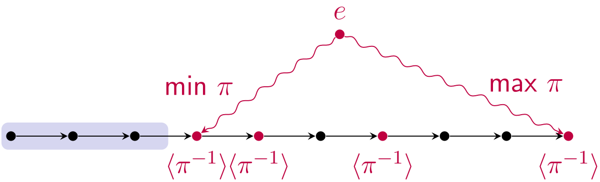

$$\pi^{\mathsf{c}} \equiv \boxed{\text{min } \pi \cdot \overset{+}{\leftarrow}}$$

Negation

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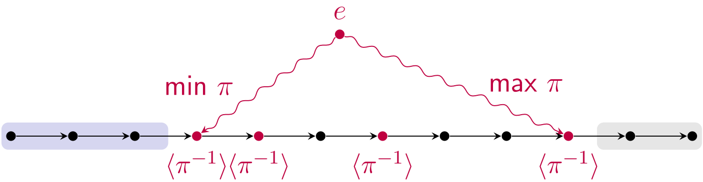

$$\pi^{c} \equiv \boxed{\text{min } \pi \cdot \xleftarrow{+}} \cup \boxed{\text{max } \pi \cdot \xrightarrow{+}}$$

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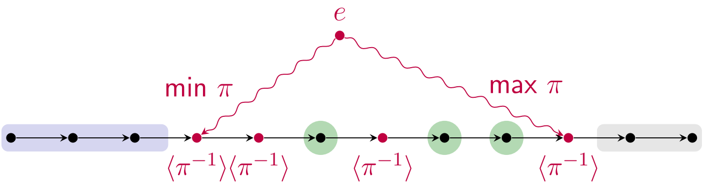

$$\pi^{\mathsf{c}} \equiv \boxed{\text{min } \pi \cdot \xleftarrow{+}} \cup \boxed{\text{max } \pi \cdot \xrightarrow{+}} \cup \boxed{\text{min } \pi \cdot \xrightarrow{+} \cdot \{\neg \langle\pi^{-1}\rangle\}?}$$

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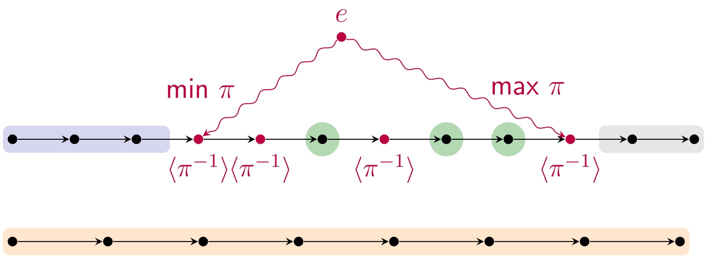

$$\pi^{\mathsf{c}} \equiv \boxed{\text{min } \pi \cdot \xleftarrow{+}} \cup \boxed{\text{max } \pi \cdot \xrightarrow{+}} \cup \boxed{\text{min } \pi \cdot \xrightarrow{+} \cdot \{\neg \langle \pi^{-1} \rangle\}?}$$

$$\cup \boxed{\bigcup_{p,q} \{\neg \langle \pi \rangle\, q\}? \cdot \mathsf{jump}_{p,q}}$$

Existential quantification

$$\exists x. \bigwedge_i \pi_i(x_i, x) \qquad \rightsquigarrow \qquad ?$$

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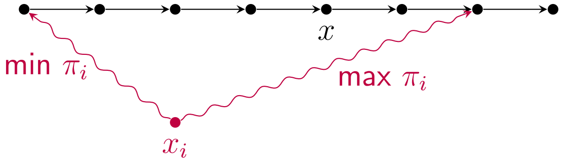

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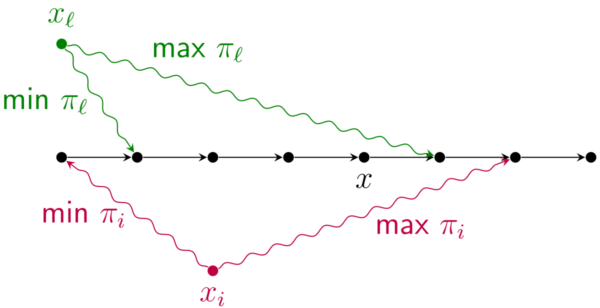

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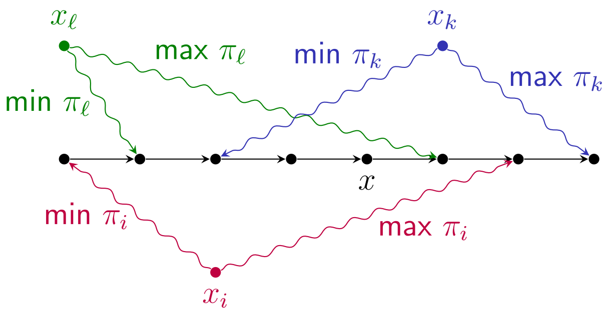

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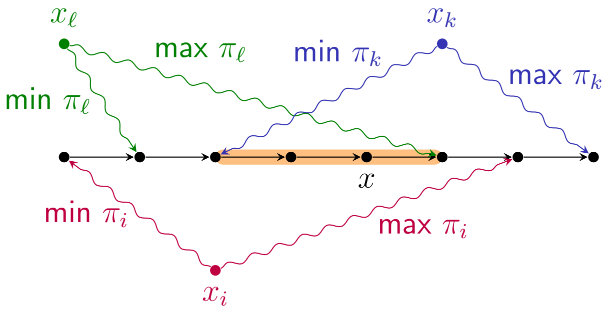

Is there an event in the intersection of the intervals that satisfies $\psi = \bigwedge_i \langle \pi_i^{-1} \rangle$?

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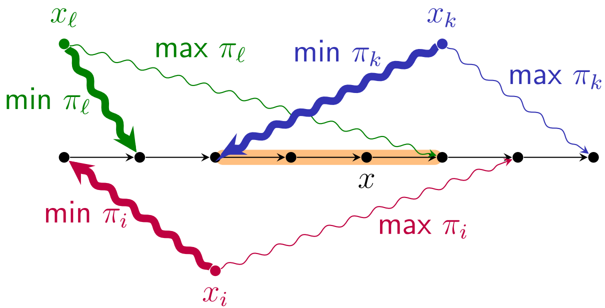

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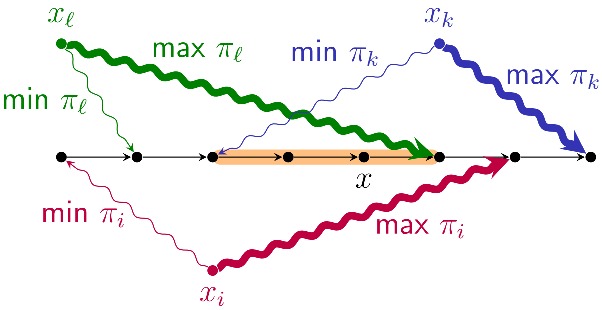

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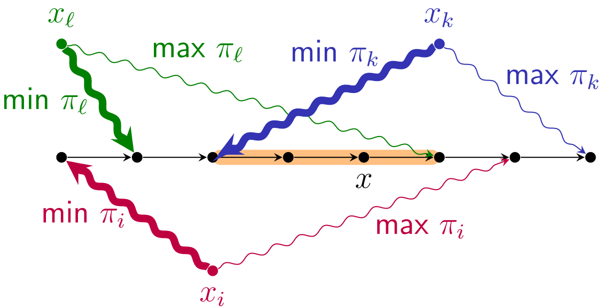

$\bigvee_k \left(\qquad \qquad \qquad \right)$

Existential quantification

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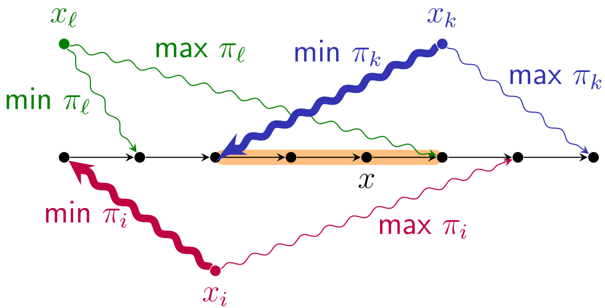

$$\bigvee_k \left(\bigwedge_i (\min \pi_i \cdot \xrightarrow{*} \cdot (\min \pi_k)^{-1})(x_i, x_k) \right)$$

Existential quantification

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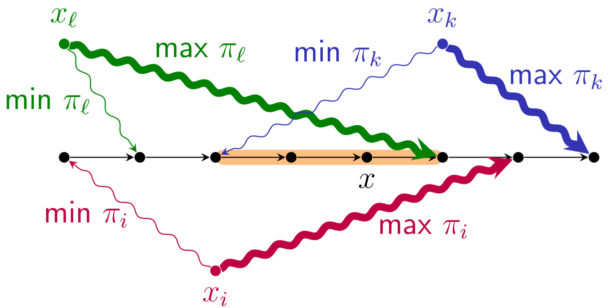

$$\bigvee_{k,\ell} \left(\bigwedge_i (\text{min } \pi_i \cdot \xrightarrow{*} \cdot (\text{min } \pi_k)^{-1})(x_i, x_k) \right)$$

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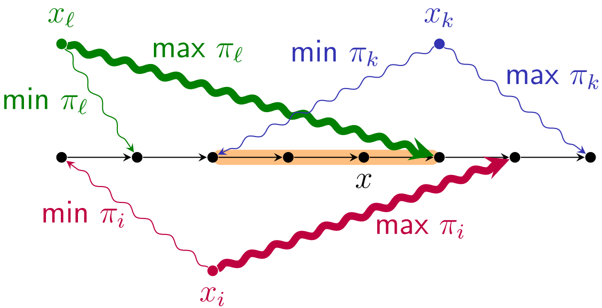

$$\bigvee_{k,\ell} \left(\begin{array}{ll} & \bigwedge_i (\text{min } \pi_i \cdot \xrightarrow{*} \cdot (\text{min } \pi_k)^{-1})(x_i, x_k) \\ \wedge & \bigwedge_i (\text{max } \pi_i \cdot \xleftarrow{*} \cdot (\text{max } \pi_\ell)^{-1})(x_i, x_\ell) \end{array} \right)$$

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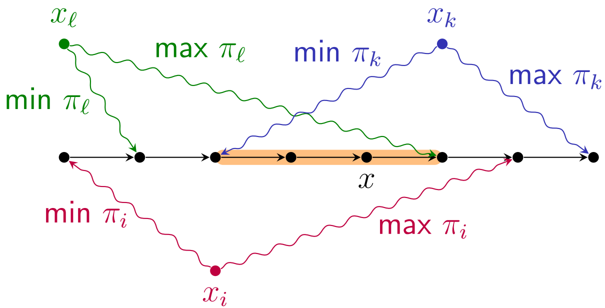

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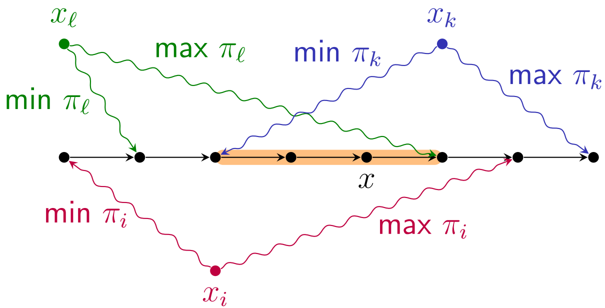

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From $\mathrm{PDL}_{\mathrm{sf}}$ to CFMs

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Theorem

Any event formula $\varphi \in \mathrm{PDL}_{\mathrm{sf}}$ can be translated into a CFM which determines for each event whether φ holds.

Proof: By induction.

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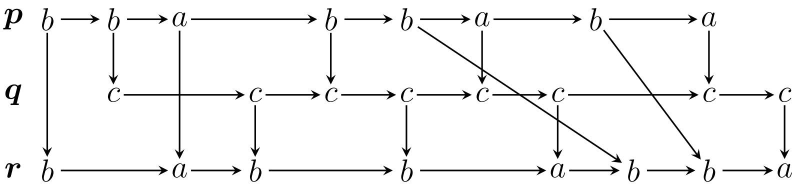

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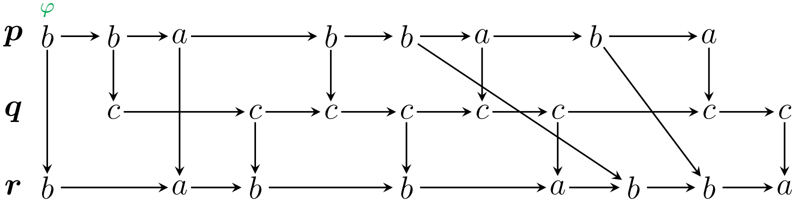

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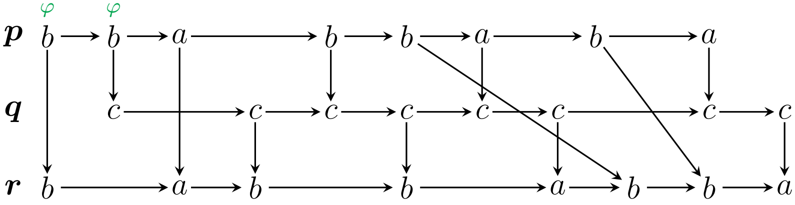

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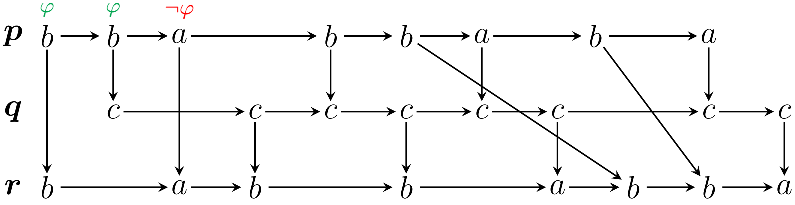

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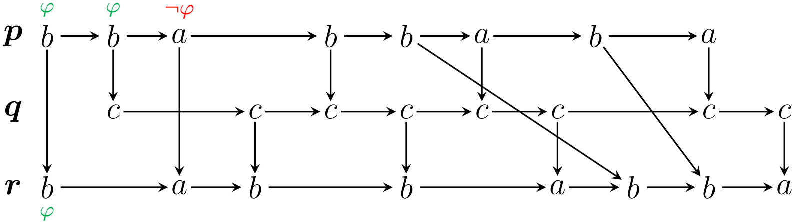

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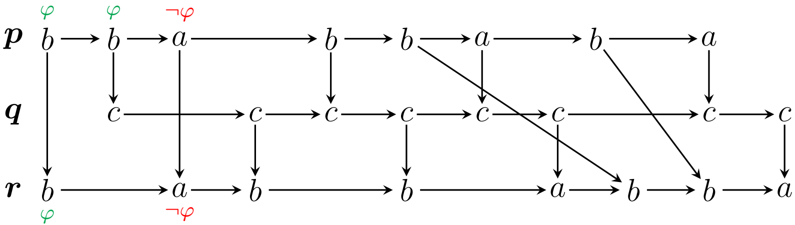

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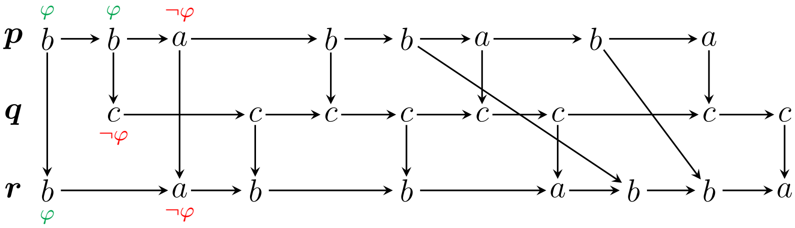

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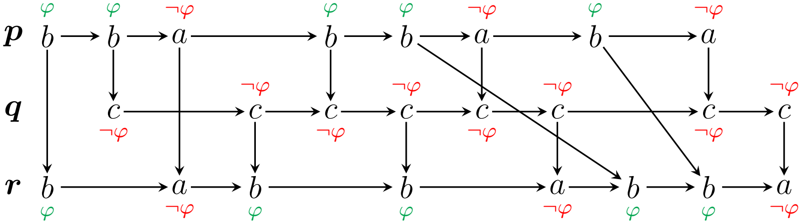

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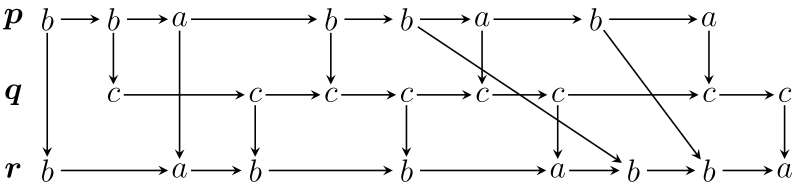

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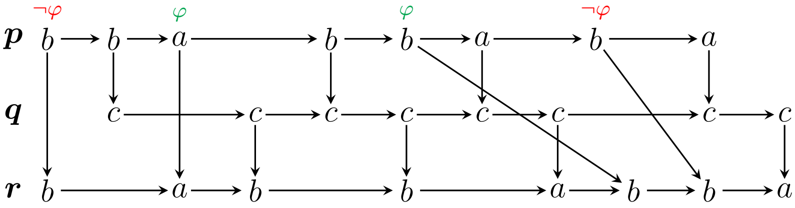

From PDL$_{\mathsf{sf}}$ to CFMs

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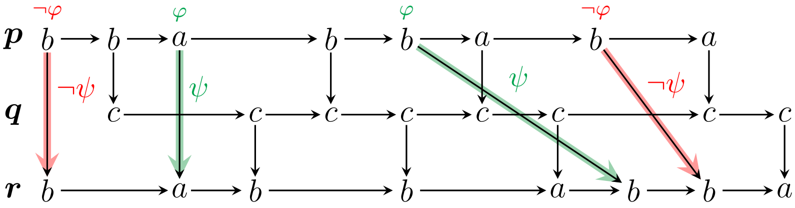

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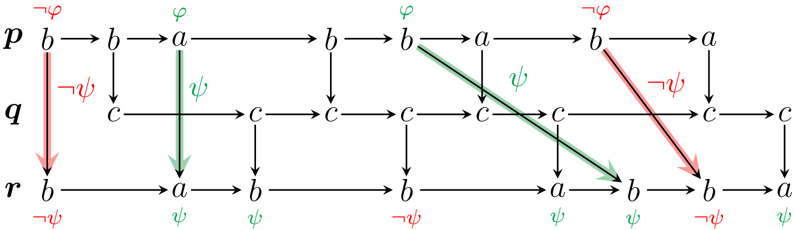

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- $\ldots$

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- ...
- Only difficult case: $\varphi = \mathrm{Loop}(\pi)$

Translation of Loop formulas

Needed: CFM that, at every event, evaluates $\mathsf{Loop}(\pi)$.

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- If $e \models \langle\pi\rangle \wedge \langle\pi^{-1}\rangle$, three possible cases:

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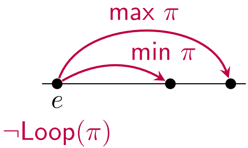

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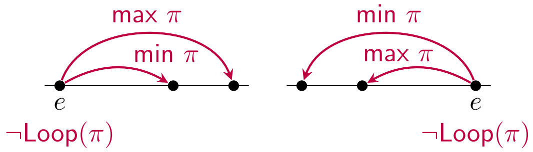
Needed: CFM that, at every event, evaluates Loop(π).

- If $e \not\models \langle\pi\rangle \wedge \langle\pi^{-1}\rangle$, then $e \not\models$ Loop(π).
- If $e \models \langle\pi\rangle \wedge \langle\pi^{-1}\rangle$, three possible cases:



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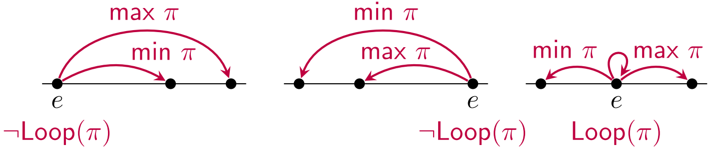

Translation of Loop formulas

Needed: CFM that, at every event, evaluates $\mathsf{Loop}(\pi)$.

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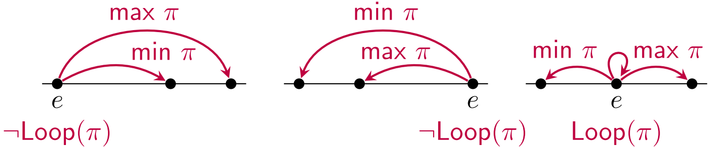

- To know which of these cases applies, it is enough to evaluate formulas $\mathsf{Loop}(\text{min } \pi')$ and $\mathsf{Loop}(\text{max } \pi')$.

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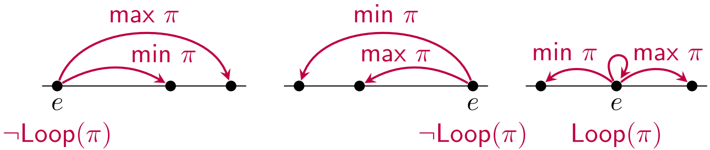


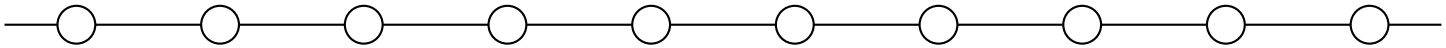
- To know which of these cases applies, it is enough to evaluate formulas Loop(min π') and Loop(max π').

1) Translate Loop(min π') to Loop(max π') into CFMs.
2) Use this to evaluate Loop(π) from left to right.

CFM for $\varphi = \mathsf{Loop}(\max \pi)$

CFM for $\varphi = \mathrm{Loop}(\max\ \pi)$

- Guess for each event whether φ holds.



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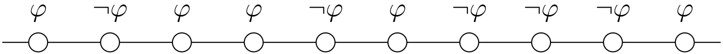

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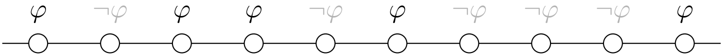

- Check positive guesses:

CFM for $\varphi = \mathsf{Loop}(\max\ \pi)$

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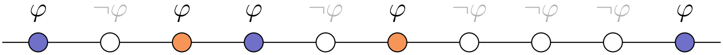

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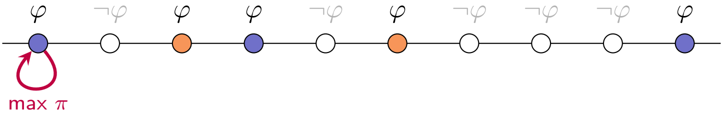

- Check positive guesses:
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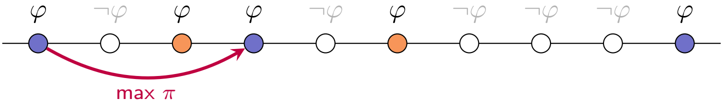

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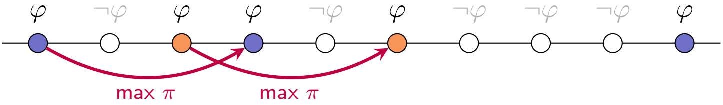

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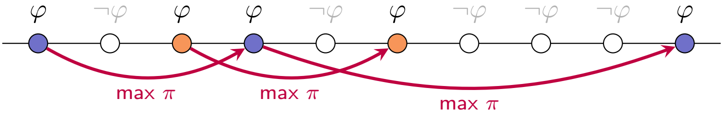

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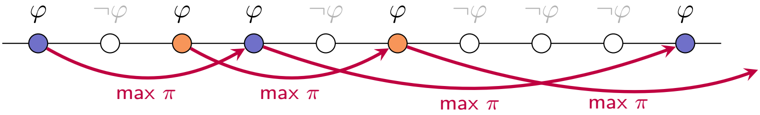

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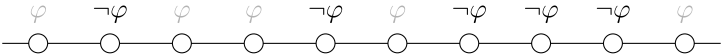

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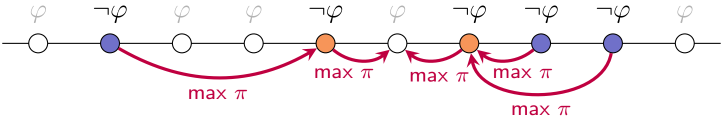

- Check positive guesses:
  - Alternately assign to φ-events colors ● or ●.
  - Check that the source and target color of $(\mathsf{max}\ \pi)$-paths are the same.
- Check negative guesses:
  - Guess a 2-coloring of the $\neg\varphi$-events.
  - Check that the source and target color of $(\mathsf{max}\ \pi)$-paths are distinct.

Summary

$$
\begin{tikzcd}
\boxed{\mathsf{FO}[\searrow, \rightarrow, \leq]} \arrow[r, "(1)"] & \boxed{\mathsf{PDL}_{\mathsf{sf}}} \arrow[r, "(2)"] \arrow[dl] & \boxed{\mathsf{CFMs} = \mathsf{EMSO}[\searrow, \rightarrow]} \\
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\end{tikzcd}
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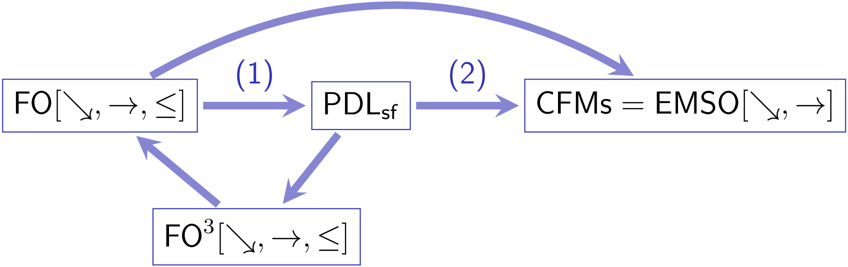

- FO sentence is a positive boolean combination of formulas $\exists x.\Phi(x)$ or $\neg\exists x.\Phi(x)$.

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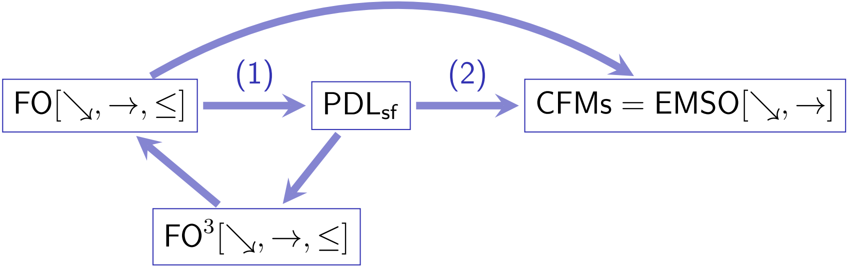

- FO sentence is a positive boolean combination of formulas $\exists x.\Phi(x)$ or $\neg\exists x.\Phi(x)$.
- Both types of formulas can be evaluated using the CFMs for $\Phi(x)$.

Conclusion

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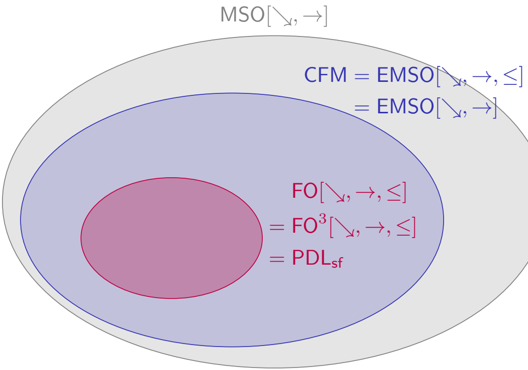

$\mathsf{MSO}[\searrow, \rightarrow]$
$\mathsf{CFM} = \mathsf{EMSO}[\searrow, \rightarrow, \leq]$
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$\mathsf{FO}[\searrow, \rightarrow, \leq]$
$= \mathsf{FO}^3[\searrow, \rightarrow, \leq]$
$= \mathsf{PDL}_{\mathsf{sf}}$

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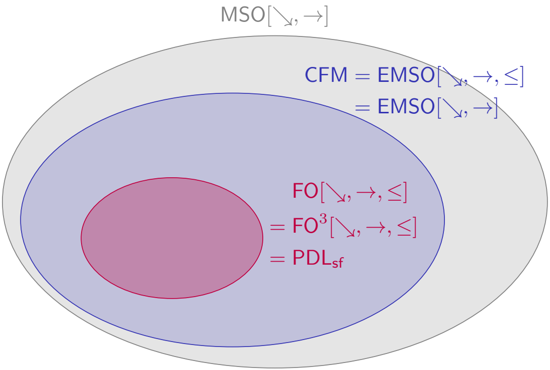

Open questions:

- Temporal logic that is expressively complete for FO?
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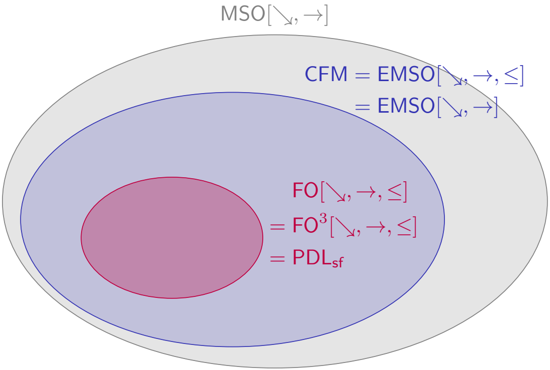

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Thank you!