Fast detection of cycles in timed automata

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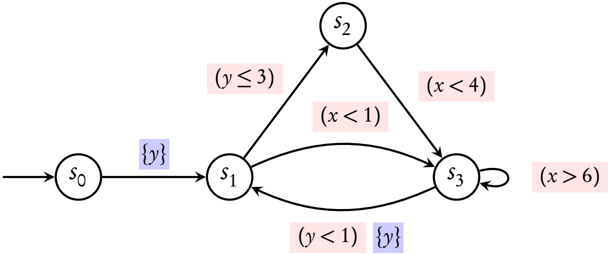
Chennai Mathematical Institute

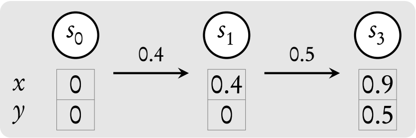
Joint work with

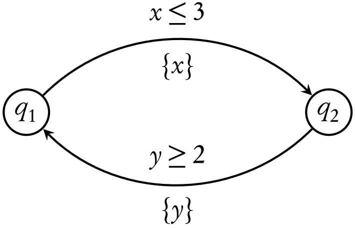
A. Deshpande *(IIT Bombay)*

T. T. Tran, F. Herbreteau, I. Walukiewicz *(LaBRI, Bordeaux)*

Timed Automata



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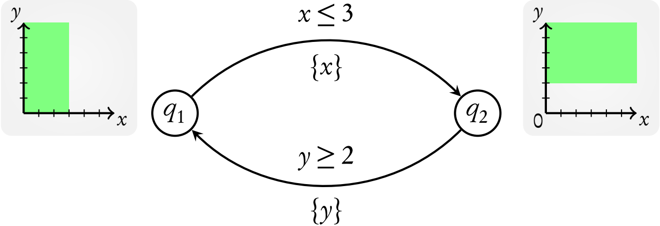
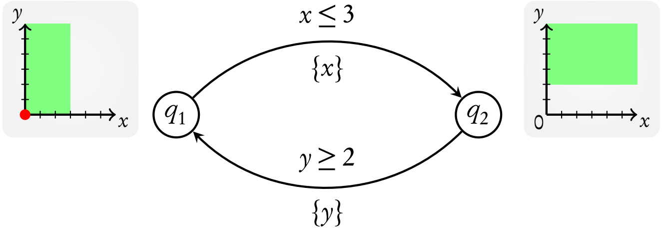
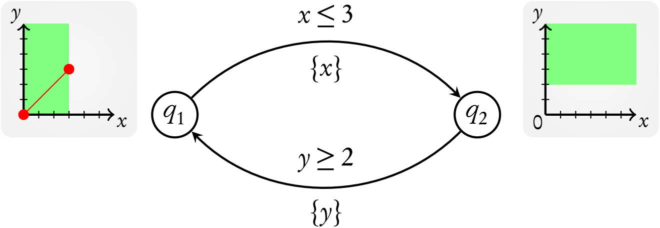
q_1
q_2
$y \geq 2$
$\{y\}$

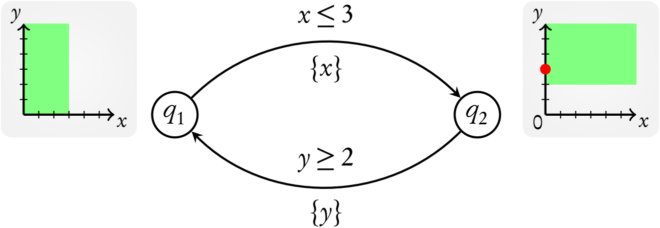

y
x
$x \leq 3$
$\{x\}$
q_1
q_2
$y \geq 2$
$\{y\}$
y
0
x



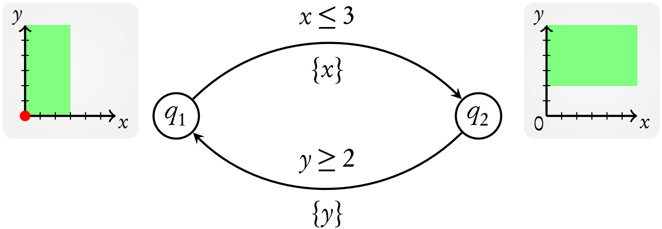
y
x
$x \leq 3$
$\{x\}$
q_1
q_2
$y \geq 2$
$\{y\}$
y
0
x



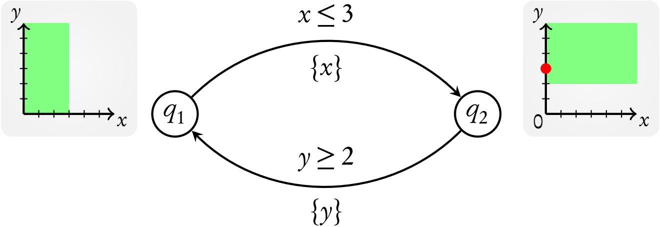
y
x
$x \leq 3$
$\{x\}$
q_1
q_2
$y \geq 2$
$\{y\}$
y
0
x



$x \leq 3$
$\{x\}$
$y \geq 2$
$\{y\}$
q_1
q_2
y
x
0



$x \leq 3$
$\{x\}$
q_1
q_2
$y \geq 2$
$\{y\}$
y
x
0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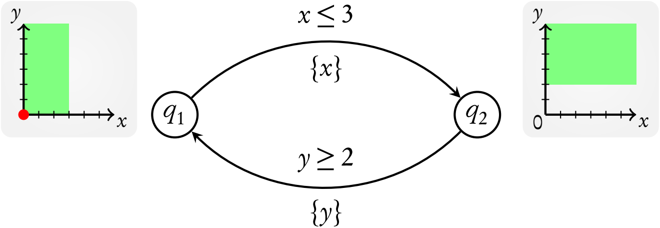
$y \geq 2$
$\{y\}$
y
x
y
0
x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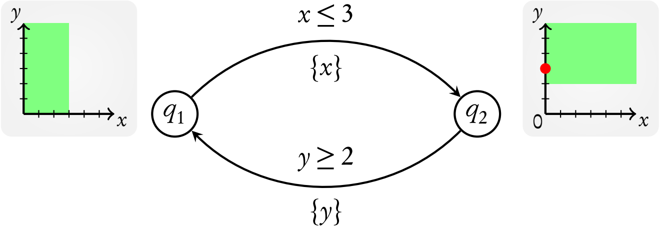
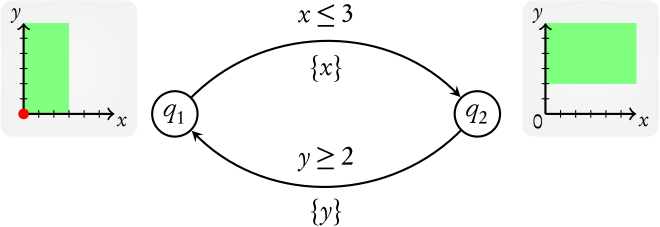
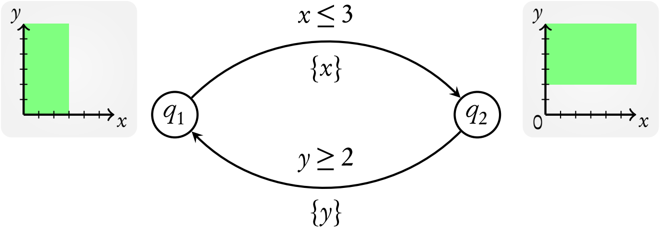
q_1
q_2
$y \geq 2$
$\{y\}$
y
x
0



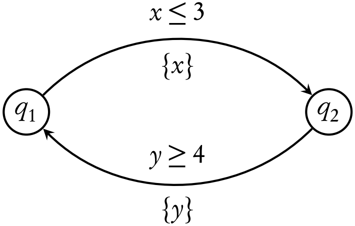
y
x
$x \leq 3$
$\{x\}$
q_1
q_2
$y \geq 2$
$\{y\}$
y
0
x

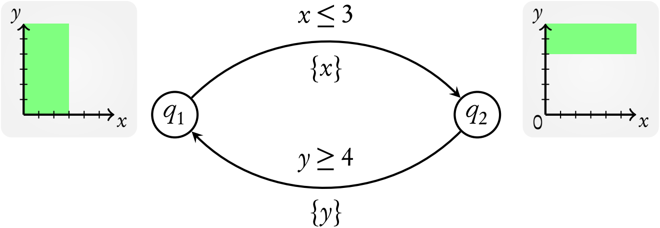


$x \leq 3$
$\{x\}$
q_1
q_2
$y \geq 2$
$\{y\}$
y
x
y
0
x

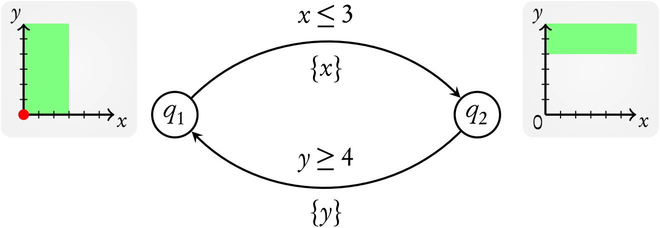


This cycle can be **iterated** infinitely often


$x \leq 3$
$\{x\}$
q_1
q_2
$y \geq 4$
$\{y\}$



y
x
x ≤ 3
{x}
q1
q2
y ≥ 4
{y}
y
0
x



y
x
$x \leq 3$
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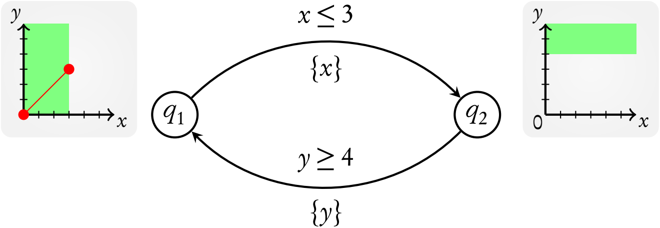
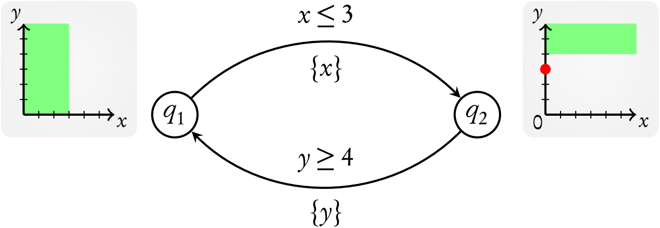
q_1
q_2
$y \geq 4$
$\{y\}$
y
0
x



y
x
x ≤ 3
{x}
q1
q2
y ≥ 4
{y}
y
0
x



y
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$x \leq 3$
$\{x\}$
q_1
q_2
$y \geq 4$
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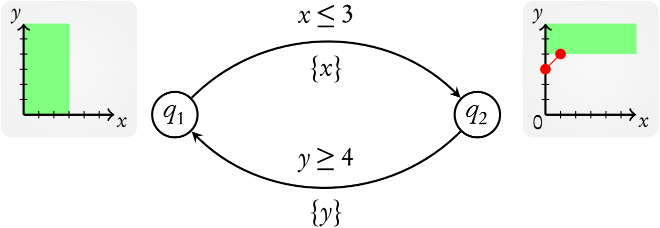
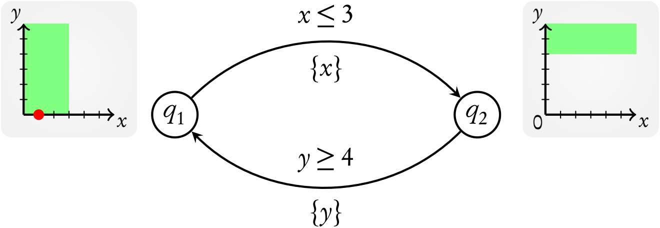
y
0
x



y
x
x ≤ 3
{x}
q1
q2
y ≥ 4
{y}
y
0
x



y
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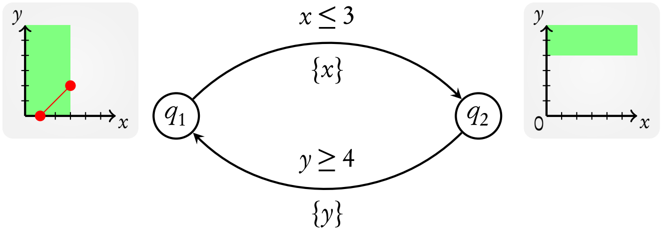
y
x
x ≤ 3
{x}
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q2
y
0
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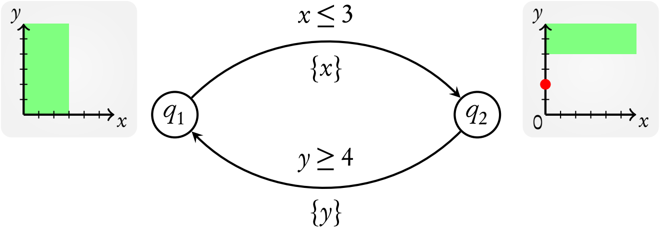
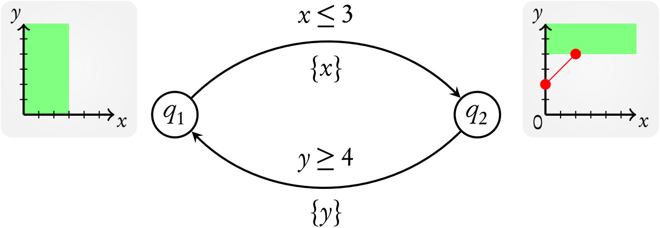
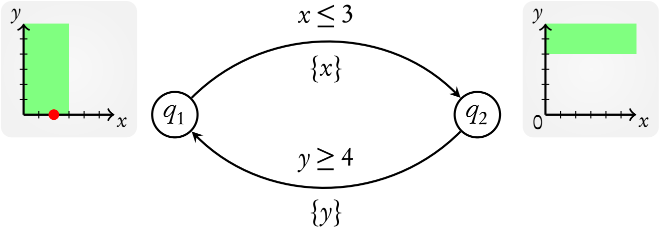
y ≥ 4
{y}



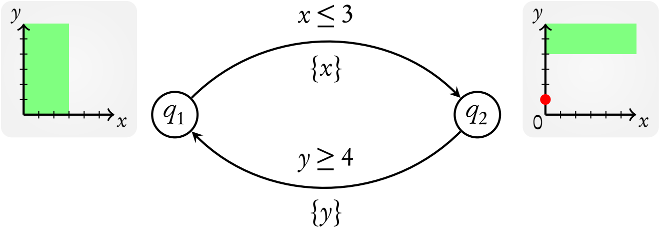
$x \leq 3$
$\{x\}$
q_1
q_2
$y \geq 4$
$\{y\}$
y
x
y
0
x



y
x
$x \leq 3$
$\{x\}$
q_1
q_2
$y \geq 4$
$\{y\}$
y
0
x



y
x
$x \le 3$
$\{x\}$
q_1
q_2
$y \ge 4$
$\{y\}$
y
0
x



y
x
$x \leq 3$
$\{x\}$
q_1
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$y \geq 4$
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y
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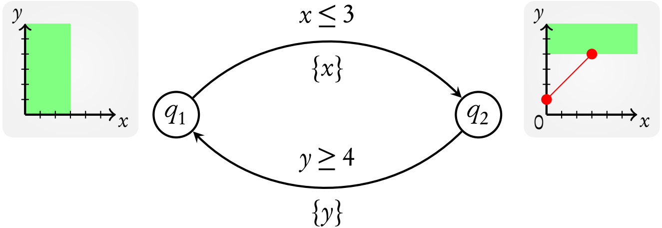
x



y
x
$x \leq 3$
$\{x\}$
q_1
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$y \geq 4$
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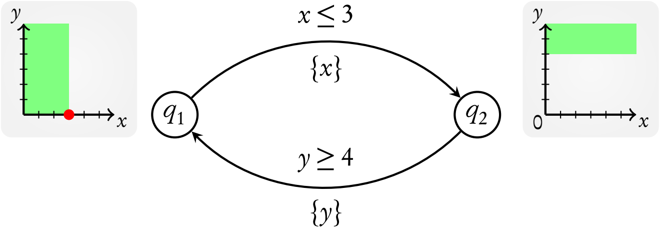
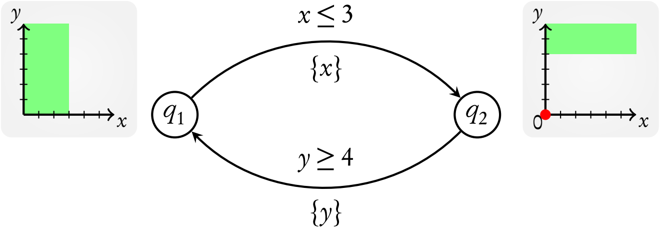
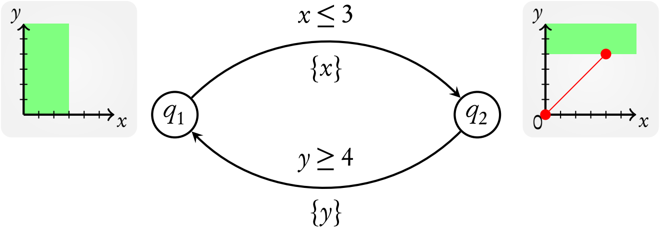
y
0
x

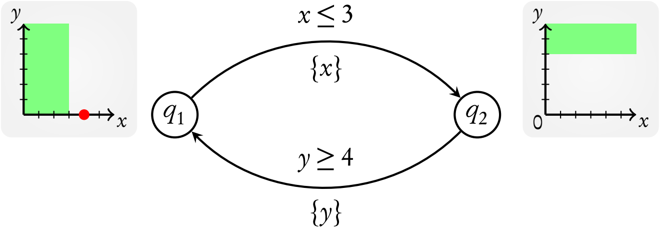

$x \leq 3$
$\{x\}$
q_1
q_2
$y \geq 4$
$\{y\}$
y
x
y
x
0



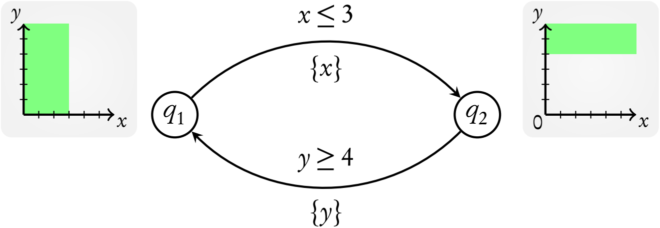
y
x
$x \leq 3$
$\{x\}$
q_1
q_2
$y \geq 4$
$\{y\}$
y
0
x



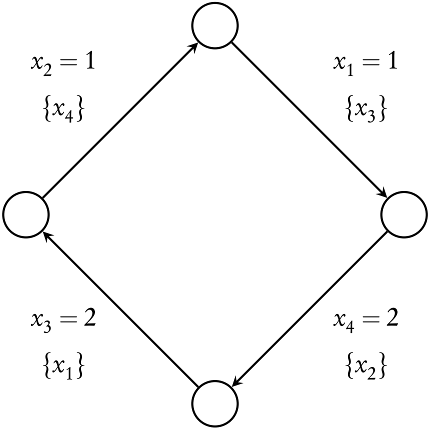
$x \leq 3$
$\{x\}$
q_1
q_2
$y \geq 4$
$\{y\}$
y
x
y
0
x



y
x
$x \leq 3$
$\{x\}$
q_1
q_2
$y \geq 4$
$\{y\}$
y
0
x

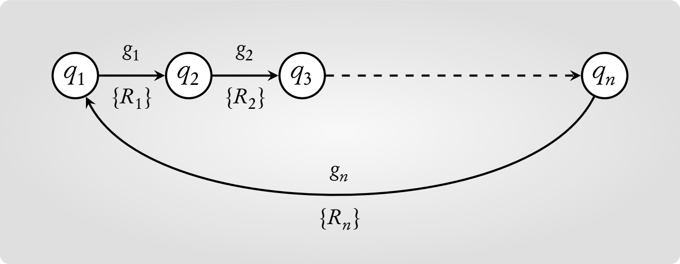


This cycle **cannot be iterated** infinitely often



This cycle **can be iterated** infinitely often

Question



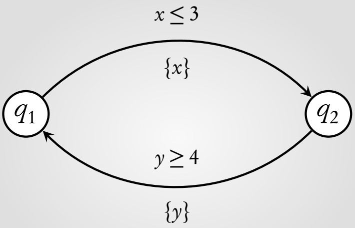
Given a cycle in the TA, is it ω-**iterable**?

Solution: find cycles in **region graph** or **zone graph**

$$(q_1, Z_1) \longrightarrow (q_1, Z_2) \dashrightarrow (q_1, \mathbf{Z}) \dashrightarrow (q_1, \mathbf{Z})$$

Solution: find cycles in **region graph** or **zone graph**

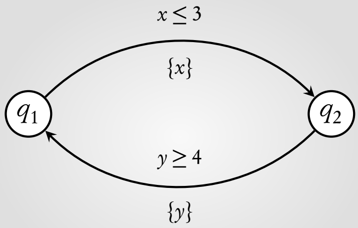
$(q_1, Z_1) \longrightarrow (q_1, Z_2) \dashrightarrow (q_1, \mathbf{Z}) \dashrightarrow (q_1, \mathbf{Z})$



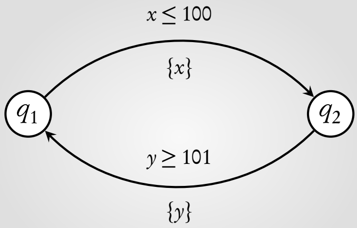
Need **4 iterations**

Solution: find cycles in **region graph** or **zone graph**

$$(q_1, Z_1) \longrightarrow (q_1, Z_2) \dashrightarrow (q_1, \mathbf{Z}) \dashrightarrow (q_1, \mathbf{Z})$$



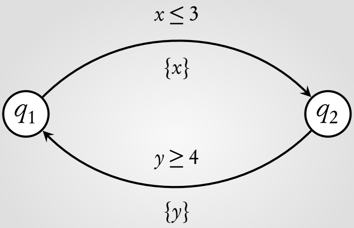
Need **4 iterations**



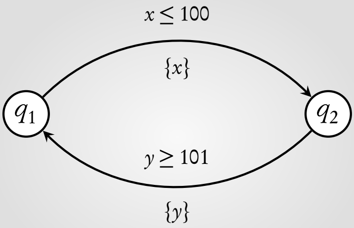
Need **101 iterations**

Solution: find cycles in **region graph** or **zone graph**

$(q_1, Z_1) \longrightarrow (q_1, Z_2) \dashrightarrow (q_1, \mathbf{Z}) \dashrightarrow (q_1, \mathbf{Z})$



Need **4 iterations**



Need **101 iterations**

Complexity: $\mathcal{O}(\, M^{n^2} \cdot |\sigma| \cdot n^3 \,)$

M: max constant

$|\sigma|$: length of cycle

n: no. of clocks

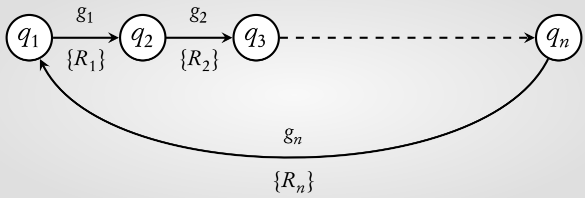
In this talk

n^2 iterations are **sufficient** to conclude ω-iterability

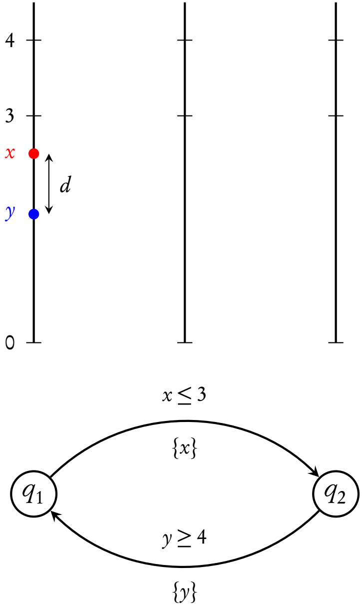
A **new algorithm** with complexity $\mathcal{O}(\ (|\sigma| + \log\ n) \cdot n^3\ )$

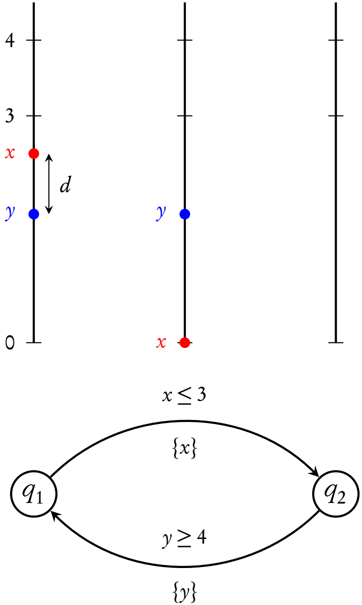
Coming next: n^2 iterations are **sufficient** to conclude ω-iterability

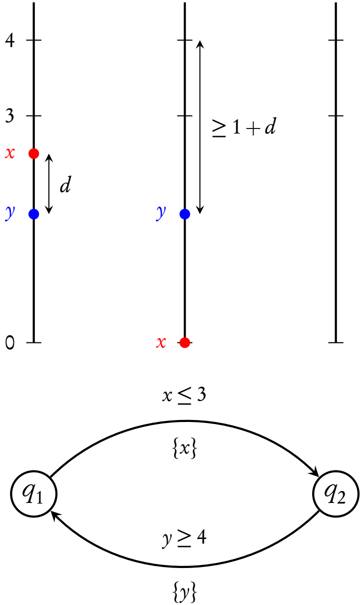
Preprocessing

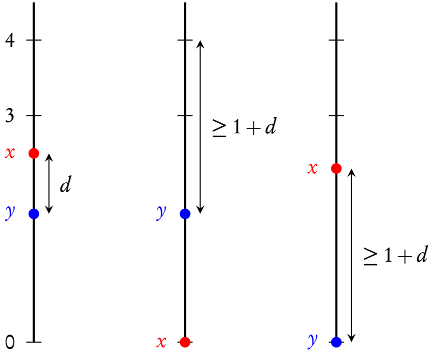


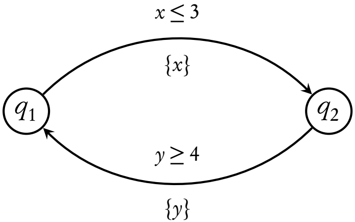
Assume the cycle **resets all clocks** at least once

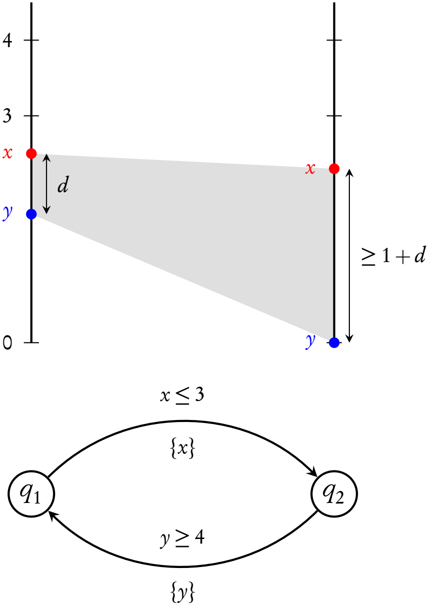

4
3
x
d
y
0
x ≤ 3
{x}
q1
q2
y ≥ 4
{y}


4
3
x
d
y
0
y
x
$x \leq 3$
$\{x\}$
q_1
q_2
$y \geq 4$
$\{y\}$

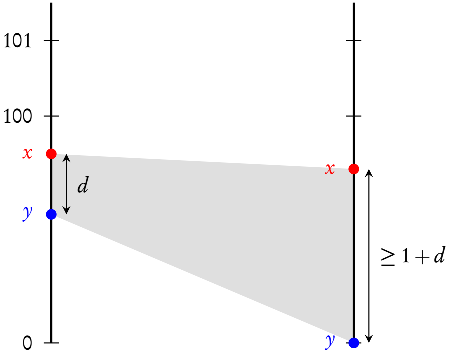

4
3
x
y
d
0
≥ 1 + d
y
x
x ≤ 3
{x}
q1
q2
y ≥ 4
{y}

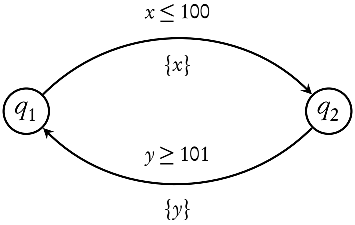

4
3
x
d
y
0
≥ 1 + d
y
x
x
≥ 1 + d
y


x ≤ 3
{x}
q1
q2
y ≥ 4
{y}

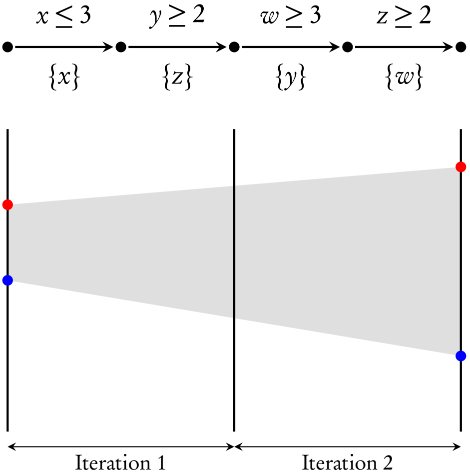


A **witness** for non-iterability





Same witness irrespective of actual **constants**



Witness might occur only **after some iterations**

- Is this witness **sufficient**?

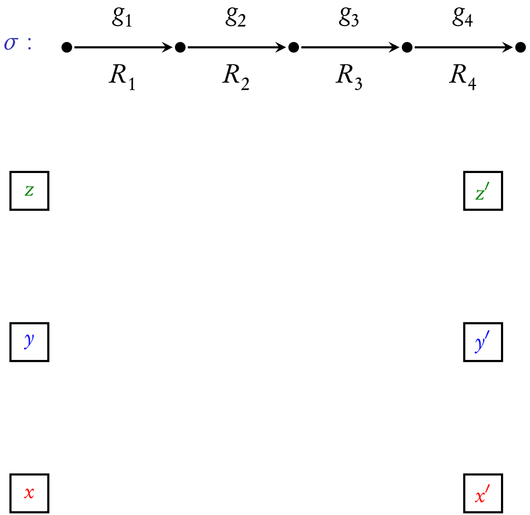
- If so, how **efficiently** can we identify it?

- Is this witness **sufficient**?

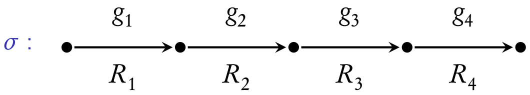
Yes

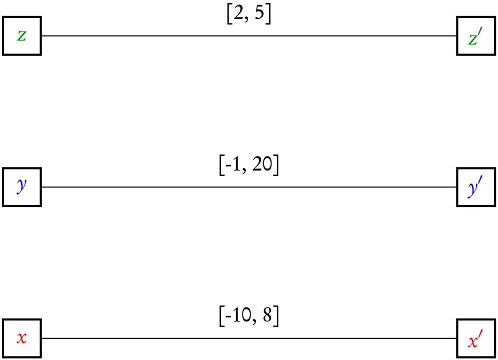
- If so, how **efficiently** can we identify it?

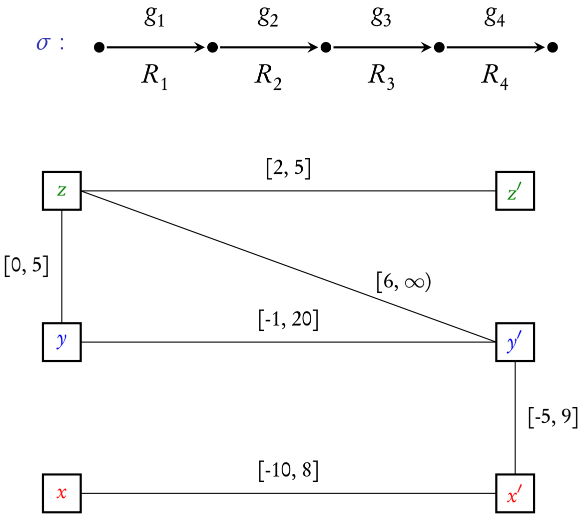
n^2 iterations; $\mathcal{O}(\ (|\sigma| + \log\ n) \cdot n^3\ )$



σ :
g_1
g_2
g_3
g_4
R_1
R_2
R_3
R_4
z
z'
y
y'
x
x'

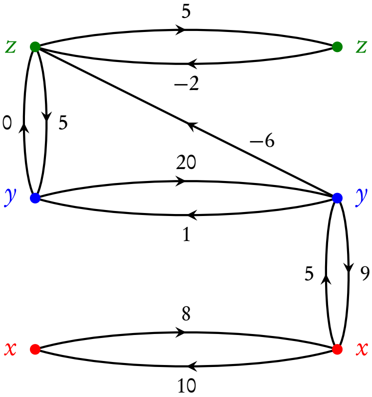

σ :
g1
g2
g3
g4
R1
R2
R3
R4


[2, 5]
z
z′
[-1, 20]
y
y′
[-10, 8]
x
x′



σ :
g1
g2
g3
g4
R1
R2
R3
R4
[2, 5]
z
z′
[0, 5]
[6, ∞)
[-1, 20]
y
y′
[-5, 9]
[-10, 8]
x
x′

Transition sequence σ $\rightarrow$ **transformation graph** G_σ



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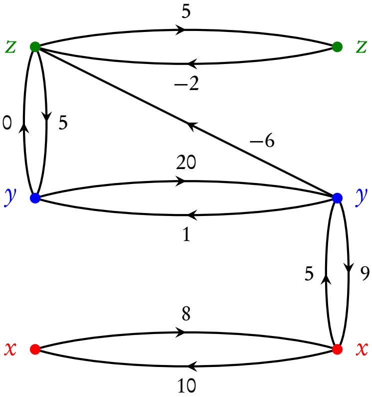
Transition sequence σ $\rightarrow$ **transformation graph** G_σ

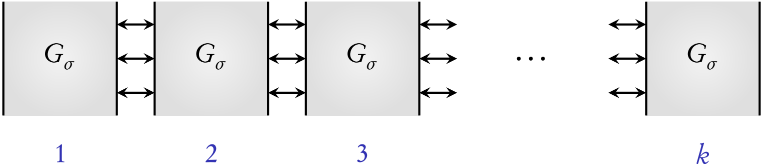


H. Comon and Y. Jurski. *Timed automata and the theory of real numbers* (CONCUR'99)

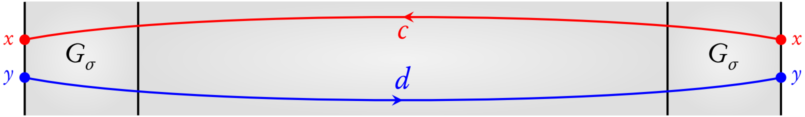
Complexity: $\mathcal{O}(\,|\sigma| \cdot n^3\,)$

To reason about k-iterations

find **shortest paths** in k-fold composition

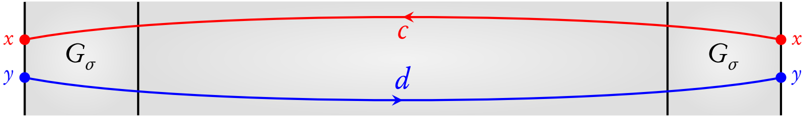


Witness



$$c + d < 0$$

Witness

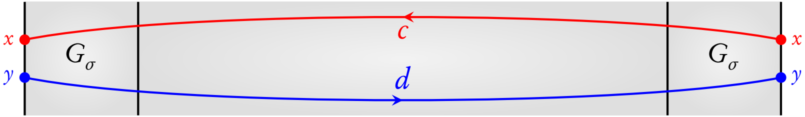


$$c + d < 0$$

Result: n^2 iterations are **sufficient**

$$\mathcal{O}(\,(\log\, n + |\sigma|) \cdot n^3\,)$$

Witness



$$c + d < 0$$

Result: n^2 iterations are **sufficient**

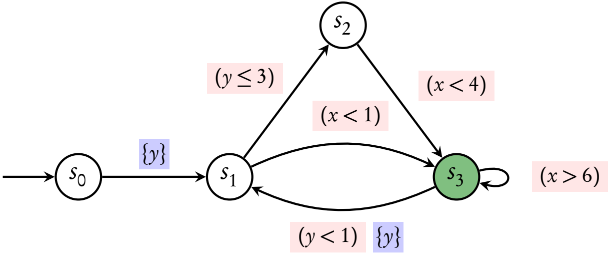
$$\mathcal{O}(\ (\log\ n + |\sigma|) \cdot n^3\ )$$

Jaubert and Reynier. *Quantitative Robustness Analysis of Timed Automata* (FOSSACS'11)

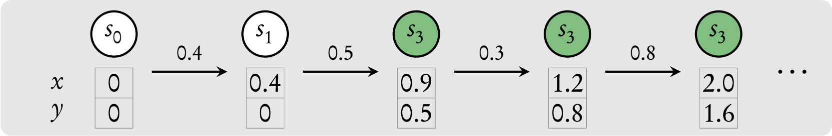
Seen so far: An ω**-iterability test** for cycles

Coming next: An **application** of the ω-iterability test

Timed Büchi Automata



Run: infinite sequence of transitions



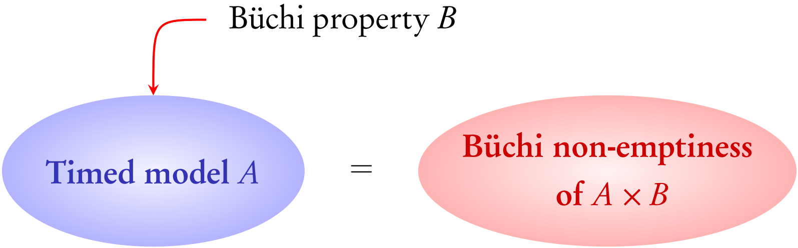
- **accepting** if infinitely often green state
- **non-Zeno** if time diverges ($\sum_{i \geq 0} \delta_i \to \infty$)

Büchi non-emptiness problem

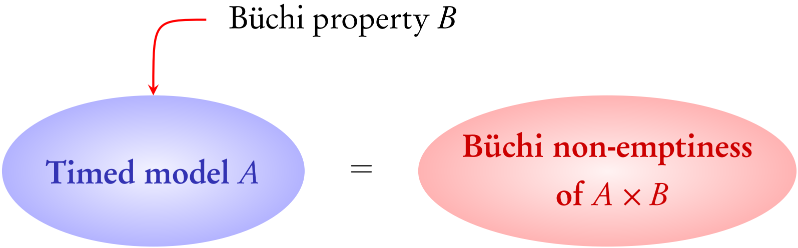
Given a (strongly non-Zeno) TBA, does it
have an accepting run

Theorem [Alur-Dill'94]

This problem is **PSPACE-complete**



Büchi property B
Timed model A
$=$
Büchi non-emptiness
of $A \times B$



Typo in standard **CSMA/CD model** revealed by Büchi test

Solution to Büchi non-emptiness is through **zone graphs**

Iterability test can be used to **accelerate** detection of accepting cycles

Model	No i-test			With i-test					
	Visited nodes			Visited nodes			Iterability checks		
	Mean	Min	Max	Mean	Min	Max	Mean	Min	Max
CSMA/CD 5	12094	11315	14677	207	9	1331	1	1	1
CSMA/CD 6	16200	12931	25861	542	10	3523	1	1	1
Fischer 4	4362	365	136263	827	7	21806	9	1	1702
Fischer 5	74924	455	1171755	14205	7	352682	27	1	1928
FDDI 9	6015	2548	16874	1964	1251	12129	25	1	73
FDDI 10	7986	2845	32019	2125	1381	18925	29	1	73
Train Gate 4	12649	281	65358	1594	4	11844	3	1	92
Train Gate 5	622207	350	1304788	86302	4	241249	34	1	442

Conclusion

- An **efficient** test for ω**-iterability**
- Application to **TBA emptiness**

- More applications of ω-iterability test
- **Better algorithms** for TBA emptiness