Programming in Haskell: Lecture 7

S P Suresh

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- `[1,2,3,1]` is a list of `Int`
- `[True,False,True]` is a list of `Bool`
- Elements of a list must be of a uniform type
- Cannot write `[1,2,True]` or `[3,'a']`

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- `[]` denotes the empty list, for all types
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- `[[3,2], [], [7,7,7]] :: [[Int]]`

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- `:` is right associative, so `1:2:3:[]` is `1:(2:(3:[]))`
- `1:[2,3] == 1:2:3:[]`, `1:2:[3] == [1,2,3]`, …all return **`True`**

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- `null l` is `True` exactly when `l` is `[]`

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- For a non-empty list `l`
  - define `f l` in terms of `head l` and `f (tail l)`

Examples

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- Compute the length of a list

```
myLength :: [Integer] -> Integer
myLength l = if null l then 0 else 1 + myLength (tail l)
```

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- Built-in function **`length`**

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- `attach l1 l2` attaches `l2` to the end of `l1`

```
attach :: [Int] -> [Int] -> [Int]
attach l1 []         = l1
attach l1 (y:ys)     = attach (addAtEnd l1 y) ys
```

Examples

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- Built-in function `++`
- `[3,2,4] ++ [5,7,6]` is `[3,2,4,5,7,6]`

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- What if `n >= length l`?
- What if `n < 0`?

Example: valueAtPosition

- Handling the problem cases:

```
valueAtPosition n l
    | null l           = error "Empty list"
    | n < 0            = error "Negative index"
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- `f n l` will be called only when `l` is non-empty and `0 <= n <= length l - 1`
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- `error` prints an error message and aborts (matches any type)

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- Need to "peel off" applications of the `:` operator
- Arrays, in other languages, allow constant-time access to any position

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  - `[2,5..19] ---> [2,5,8,11,14,17]`
- Can have descending sequences
  - `[8,7..5] ---> [8,7,6,5]`
  - `[12,8..(-9)] ---> [12,8,4,0,-4,-8]`

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myReverse []     = []
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- Number of steps is proportional to n^2, where n is the length
- Built-in function **`reverse`** is smarter

Built-in functions on lists

```
head (x:xs)      = x

tail (x:xs)      = xs

length []        = 0
length (x:xs)    = 1 + length xs

sum []           = 0
sum (x:xs)       = x + sum xs
```

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- `last` returns the last element of a list
- Undefined for the empty list
- Possible implementations:

```
init [x]       = []
init (x:xs)    = x:init xs

last [x]       = x
last (x:xs)    = last xs
```

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- **`take`** `n l` returns the first `n` elements of `l`
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- `take n l` returns the first `n` elements of `l`
- `drop n l` returns all but the first `n` elements of `l`
- `take n l ++ drop n l == l`

```
take _ []                  = []
take n (x:xs) | n <= 0     = []
              | otherwise = x:take (n-1) xs

drop _ []                  = []
drop n (x:xs) | n <= 0     = x:xs
              | otherwise = drop (n-1) xs
```

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- The built-in **reverse** takes time proportional to n, the length of the list
- Strategy: Repeatedly extract head and place it in front of an accumulator list
- The list is automatically reversed

```
reverse l                 = revInto [] l
    where
        revInto a []      = a
        revInto a (x:xs) = revInto (x:a) xs
```