Programming in Haskell
Aug-Nov 2015

LECTURE 13

SEPTEMBER 29, 2015

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Search problems

- No closed form for the solution
- Go through a cycle of
  - expanding out possible solutions
  - undoing partial solutions when we reach a dead end
- **Backtracking**

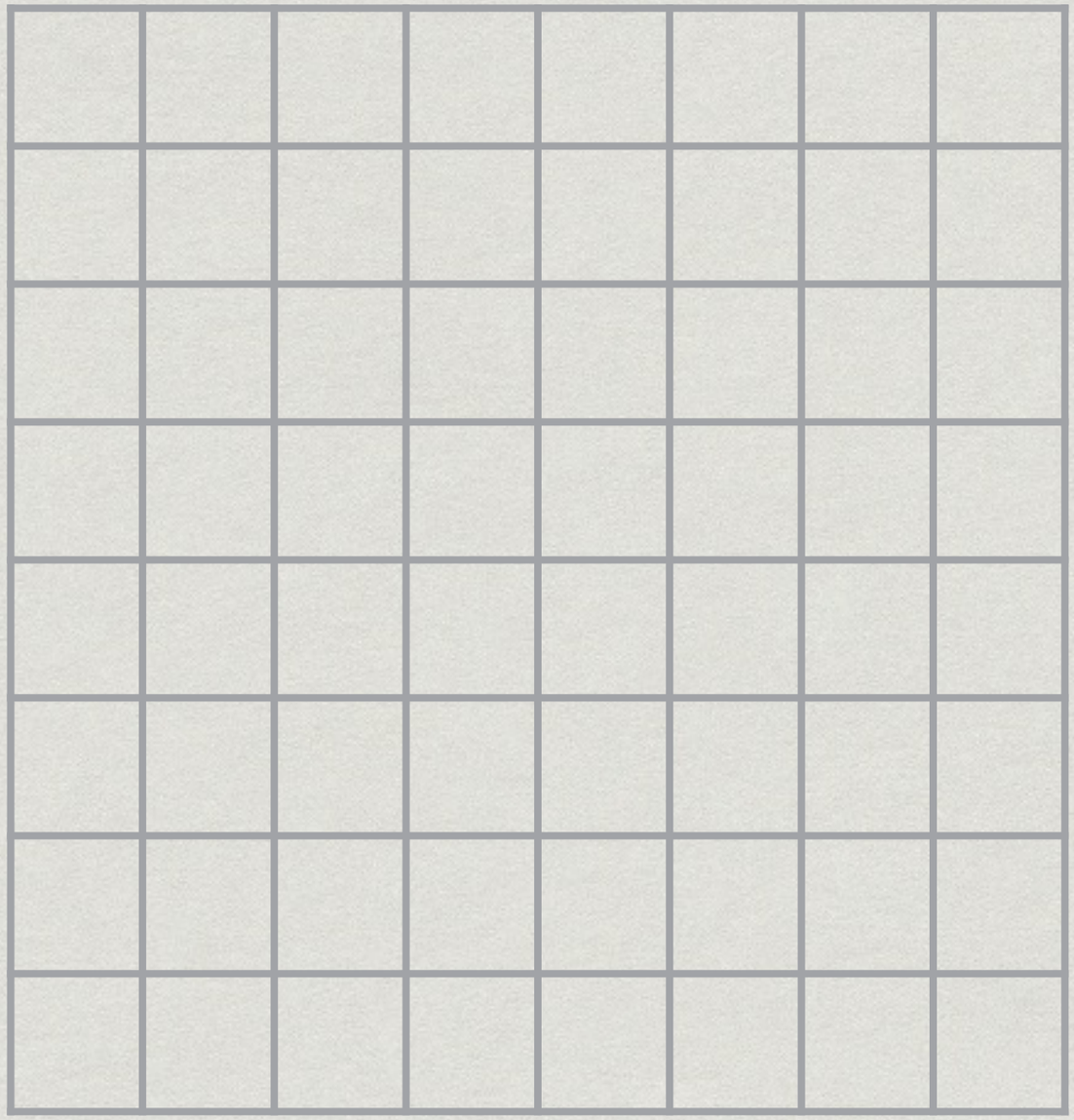
N queens

- Place N queens on N x N chessboard so that no two attack each other
  - Two queens attack each other if they are on the same row, column or diagonal
- N queens, N rows, N columns
  - Exactly one queen in each row, each column

Heuristic

- Place the first queen somewhere in the first row
- In each succeeding row, place a queen at the leftmost square that is not attacked by any of the earlier queens

Backtracking ...



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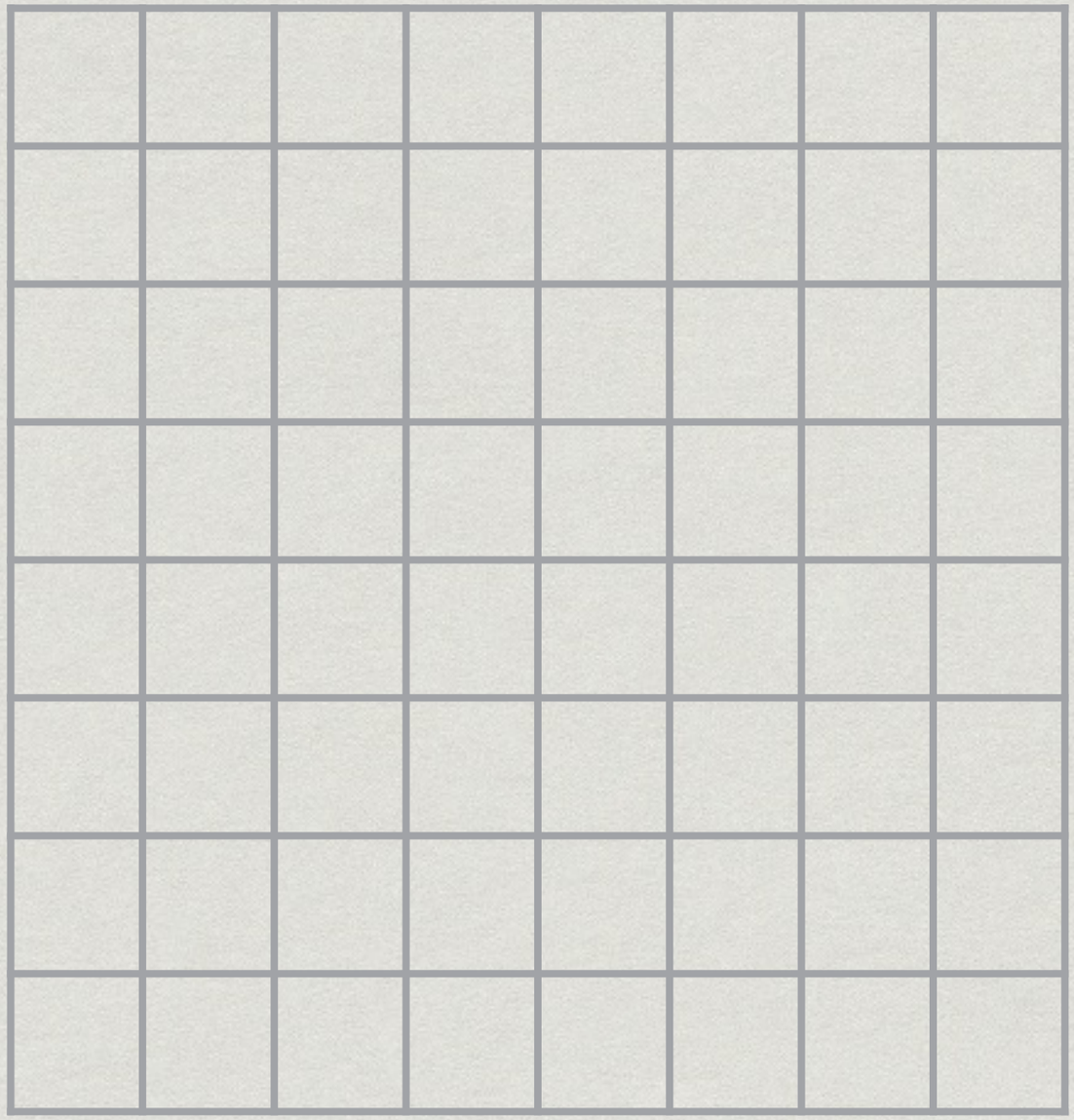

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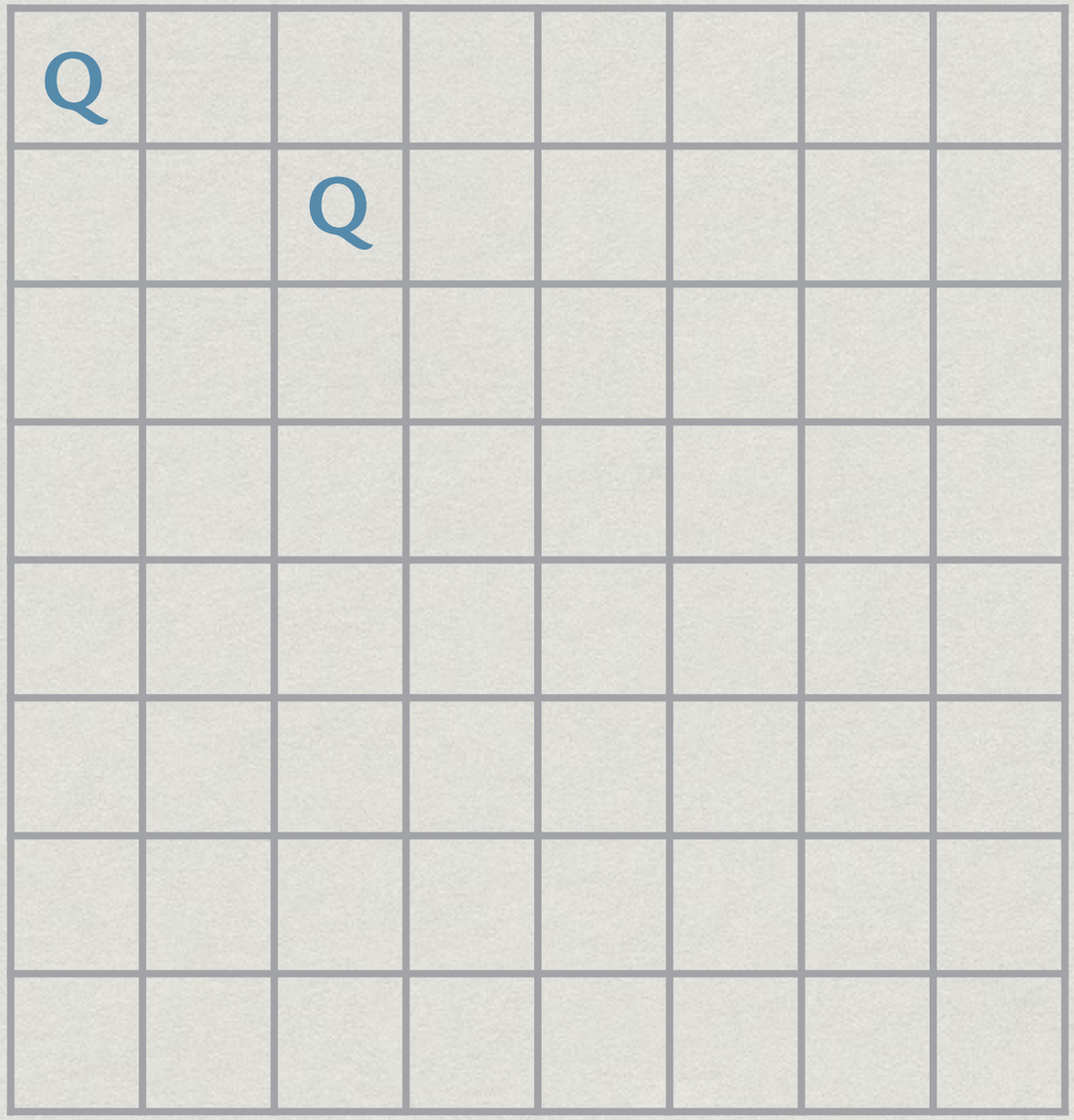

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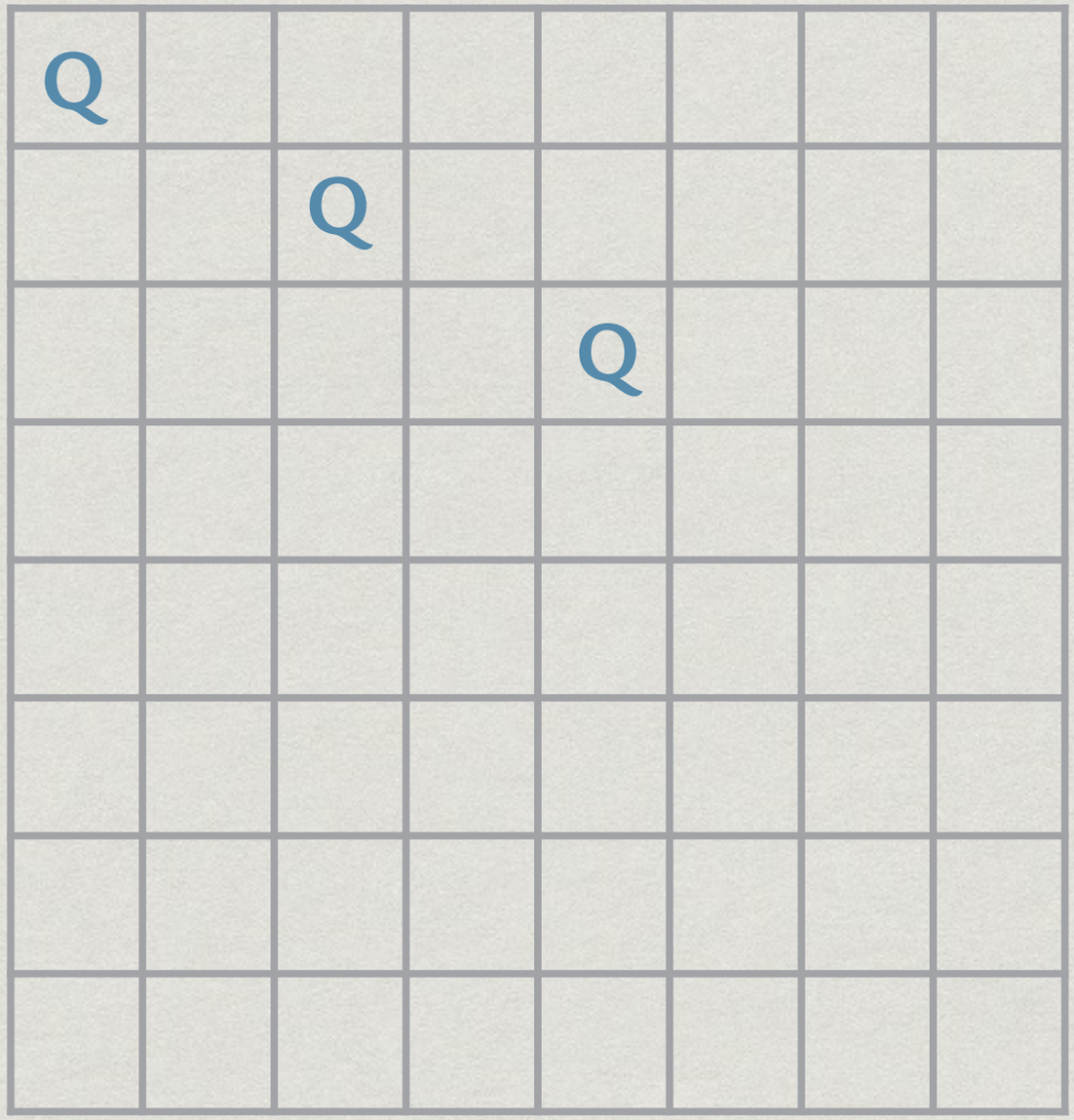

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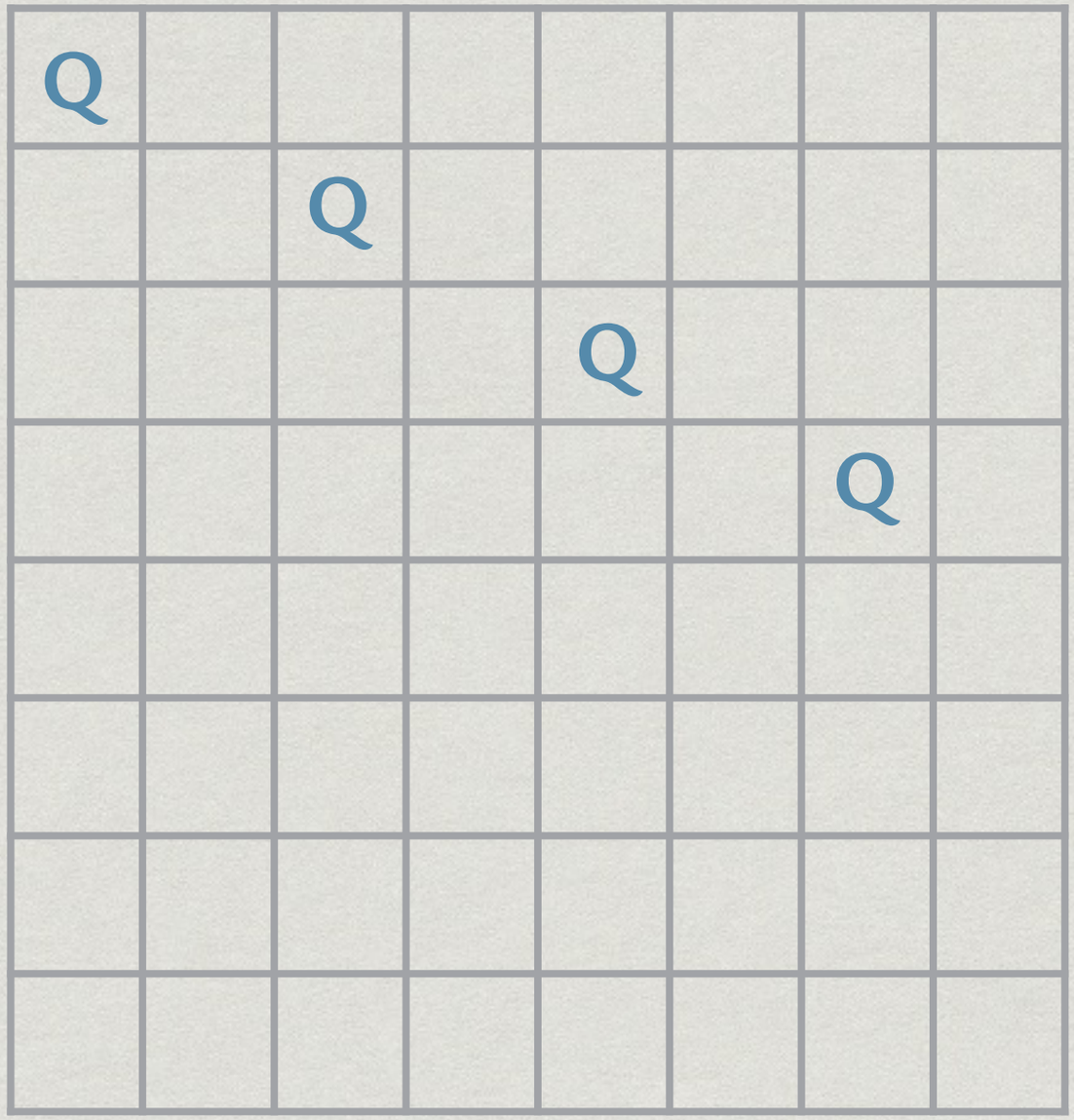

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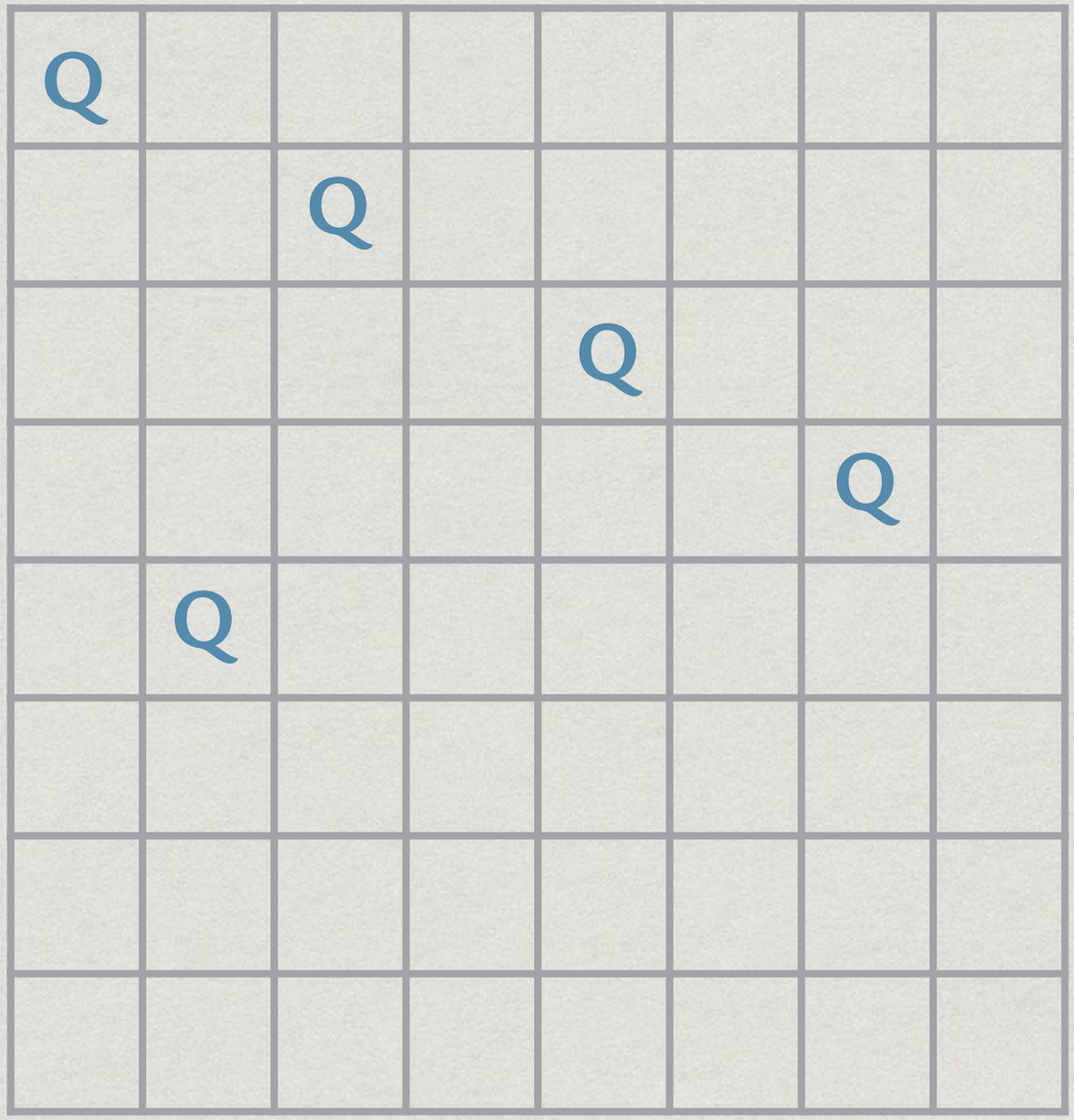

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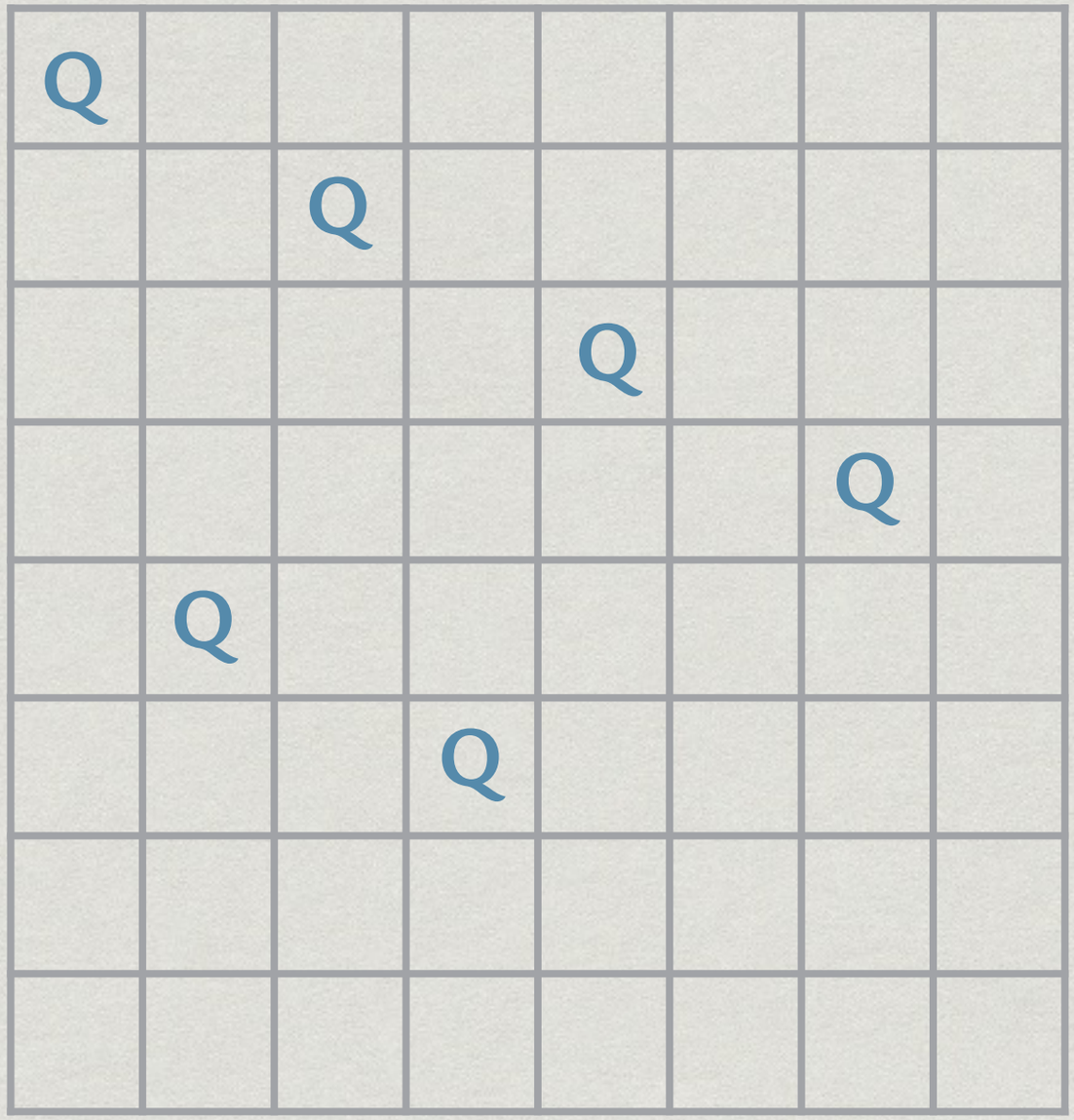

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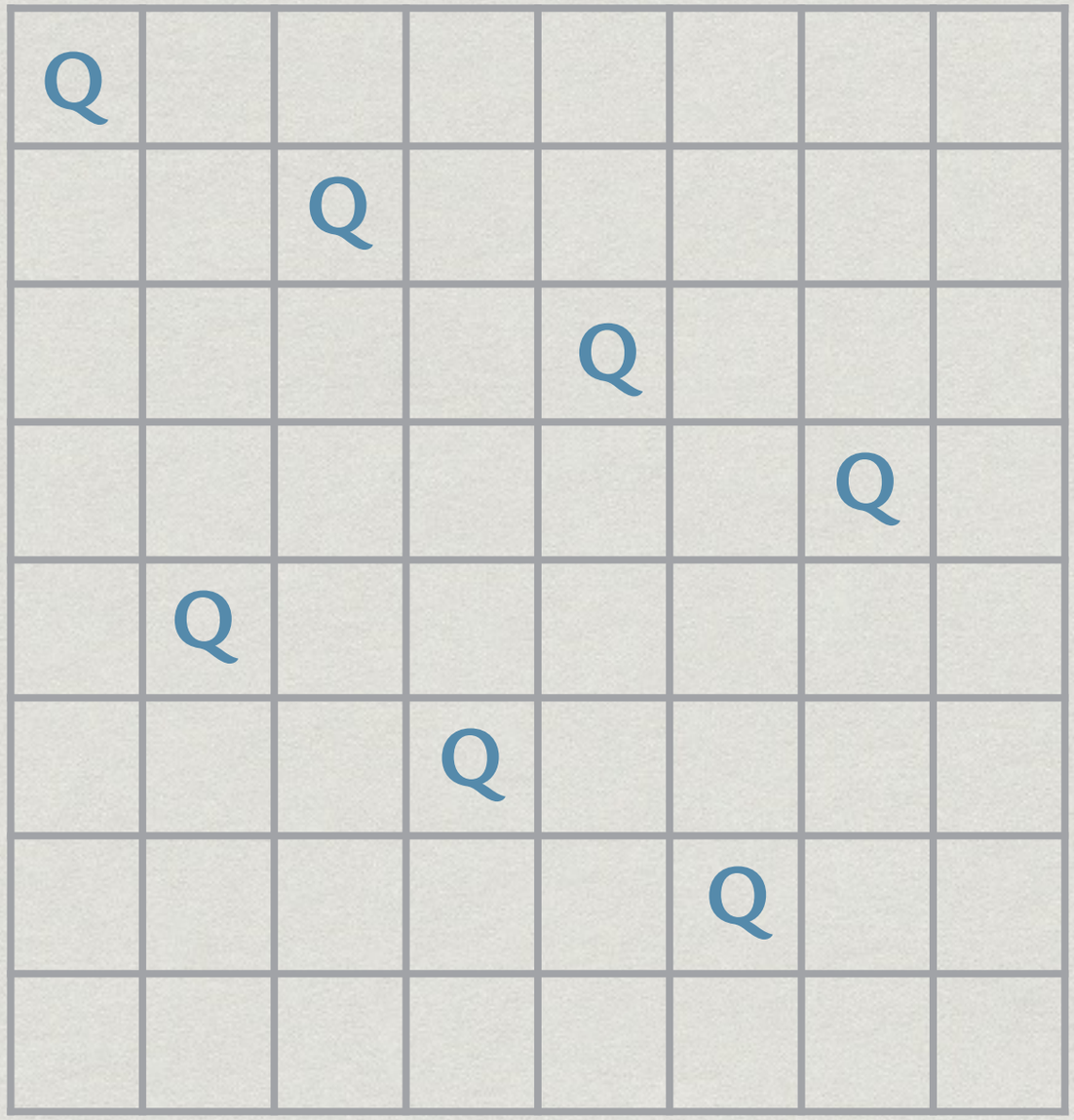

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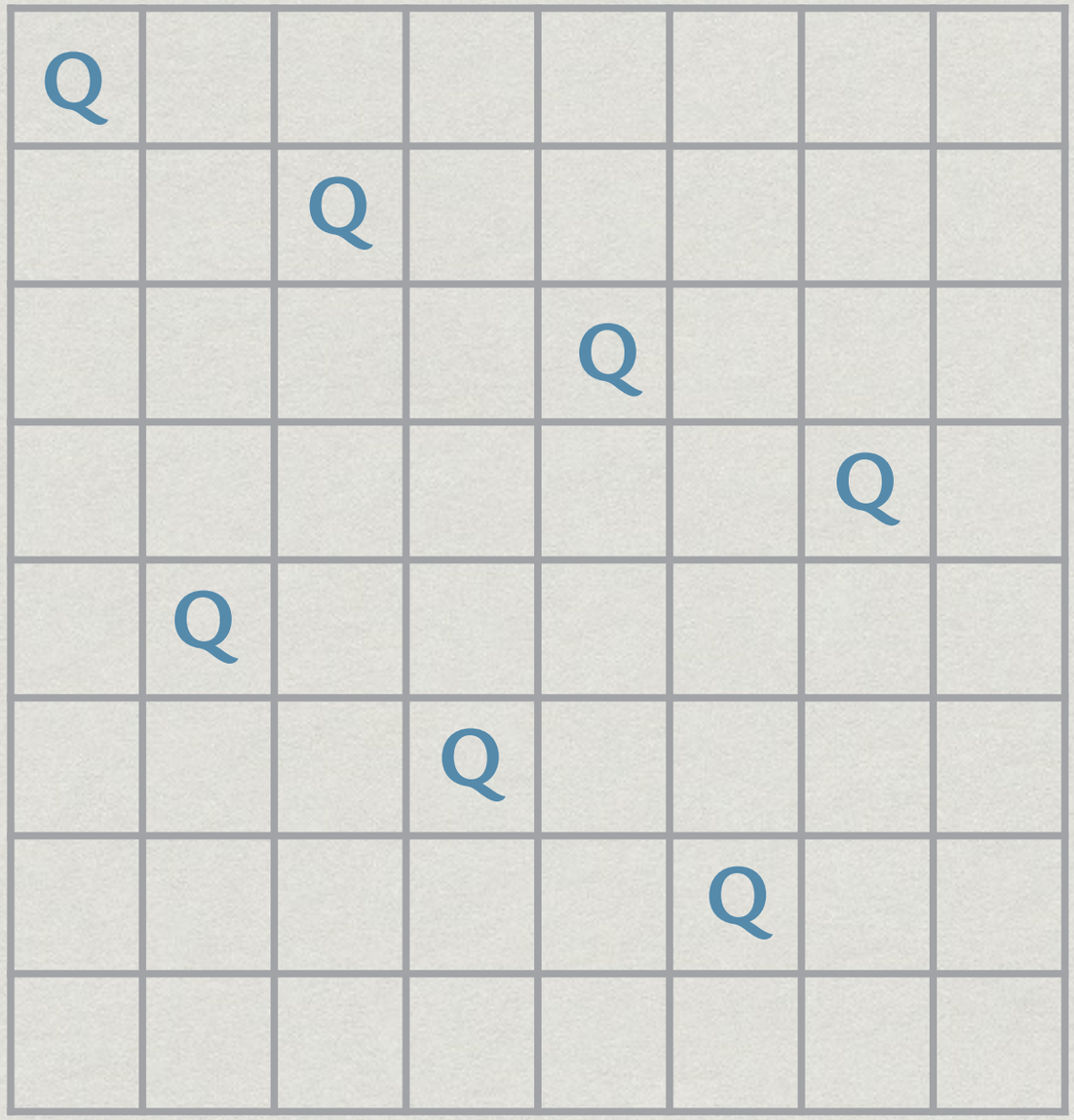

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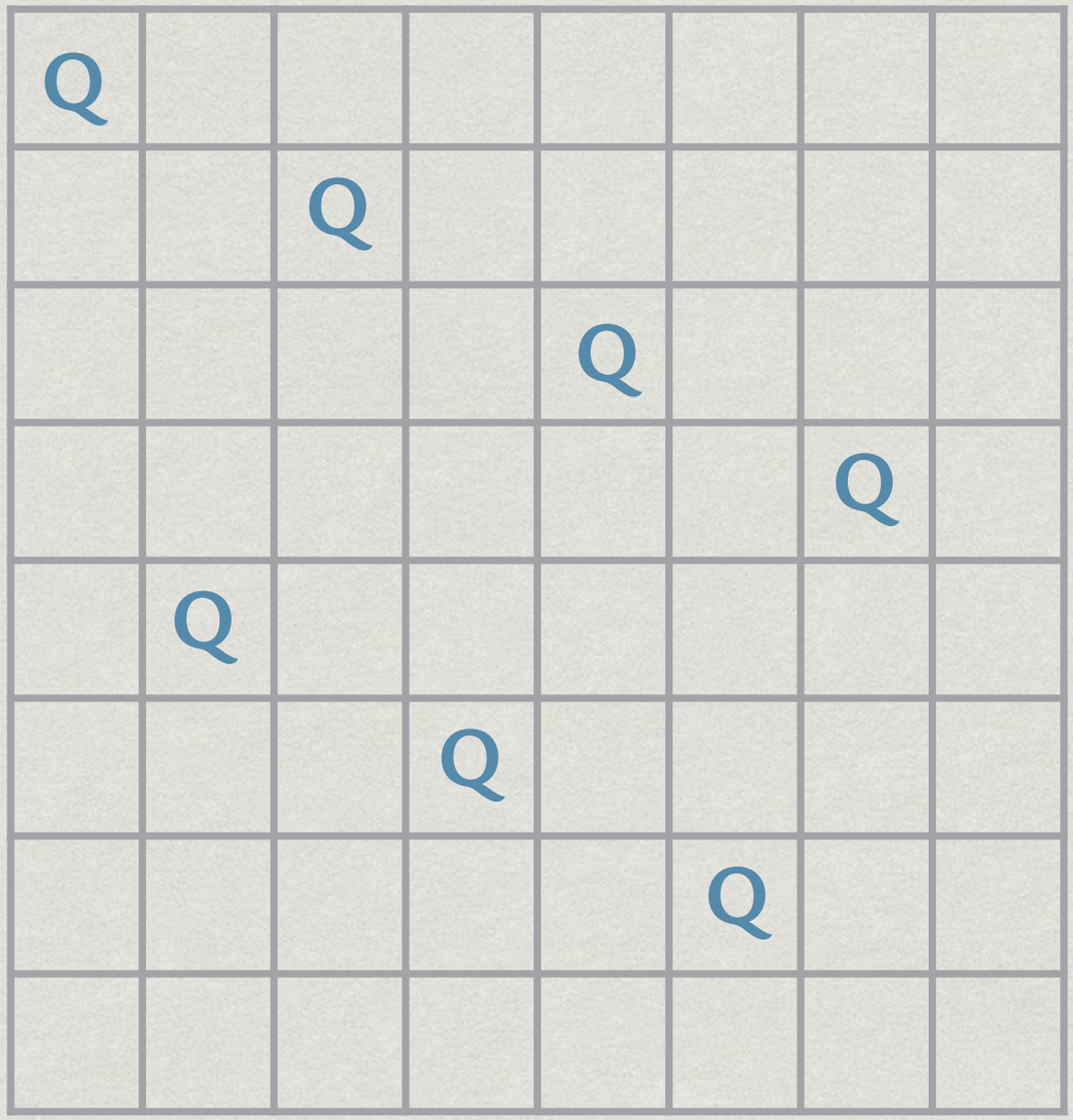

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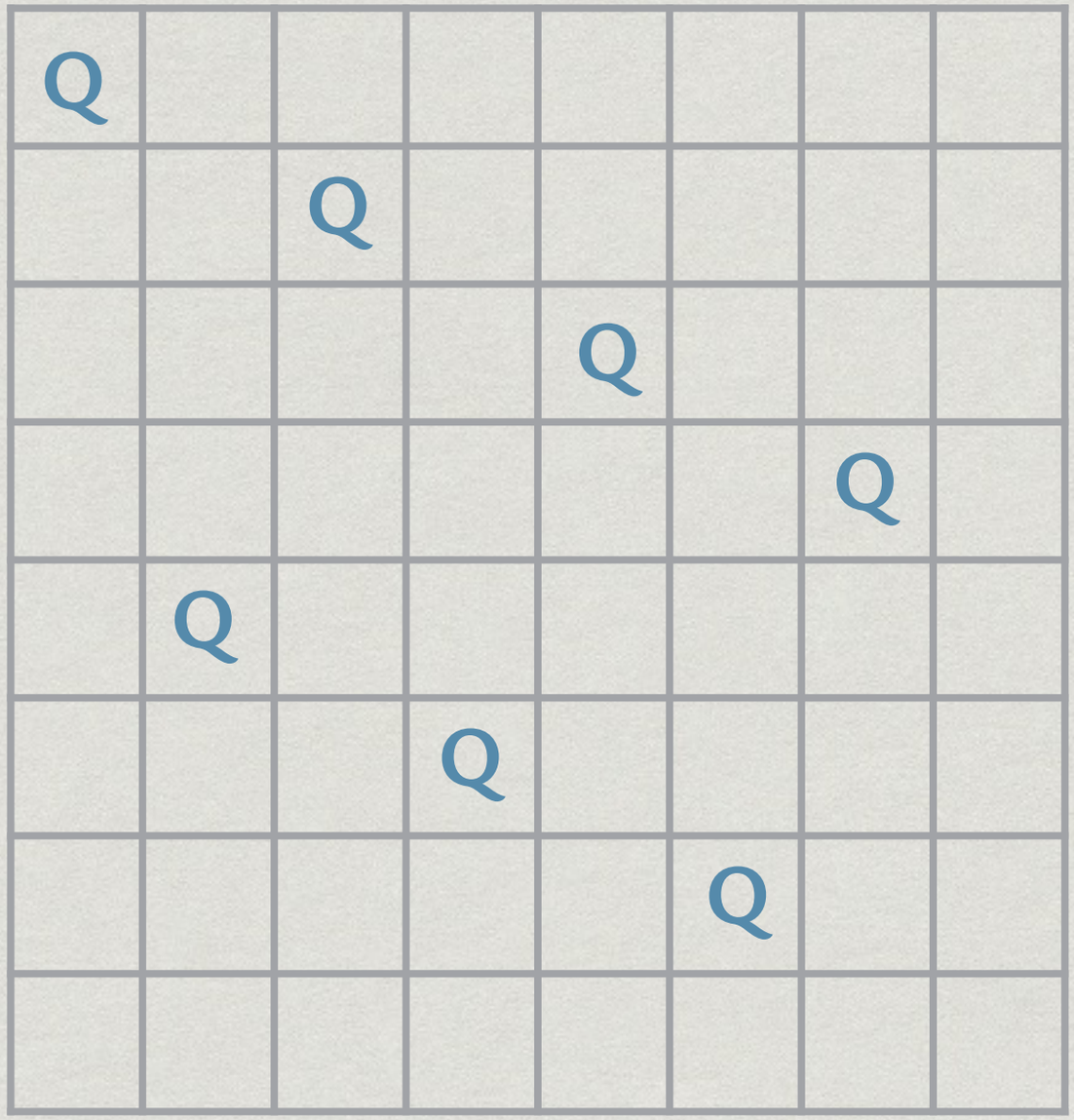
- Go back and try other positions for queen 6 …



Searching for a solution

- Arrangement of queens is a list
  - Position `i` describes queen in row `i`
  - Value is column number
  - Previous arrangement is `[0,2,4,6,1,3,5]`

Searching for a solution ...

- `extend` — function to add queen `k+1` to current list of `k` queens
  - Should not be in same column as any earlier queen
  - Should not be on same diagonals — values on a diagonal agree on `r+c` or `r-c`

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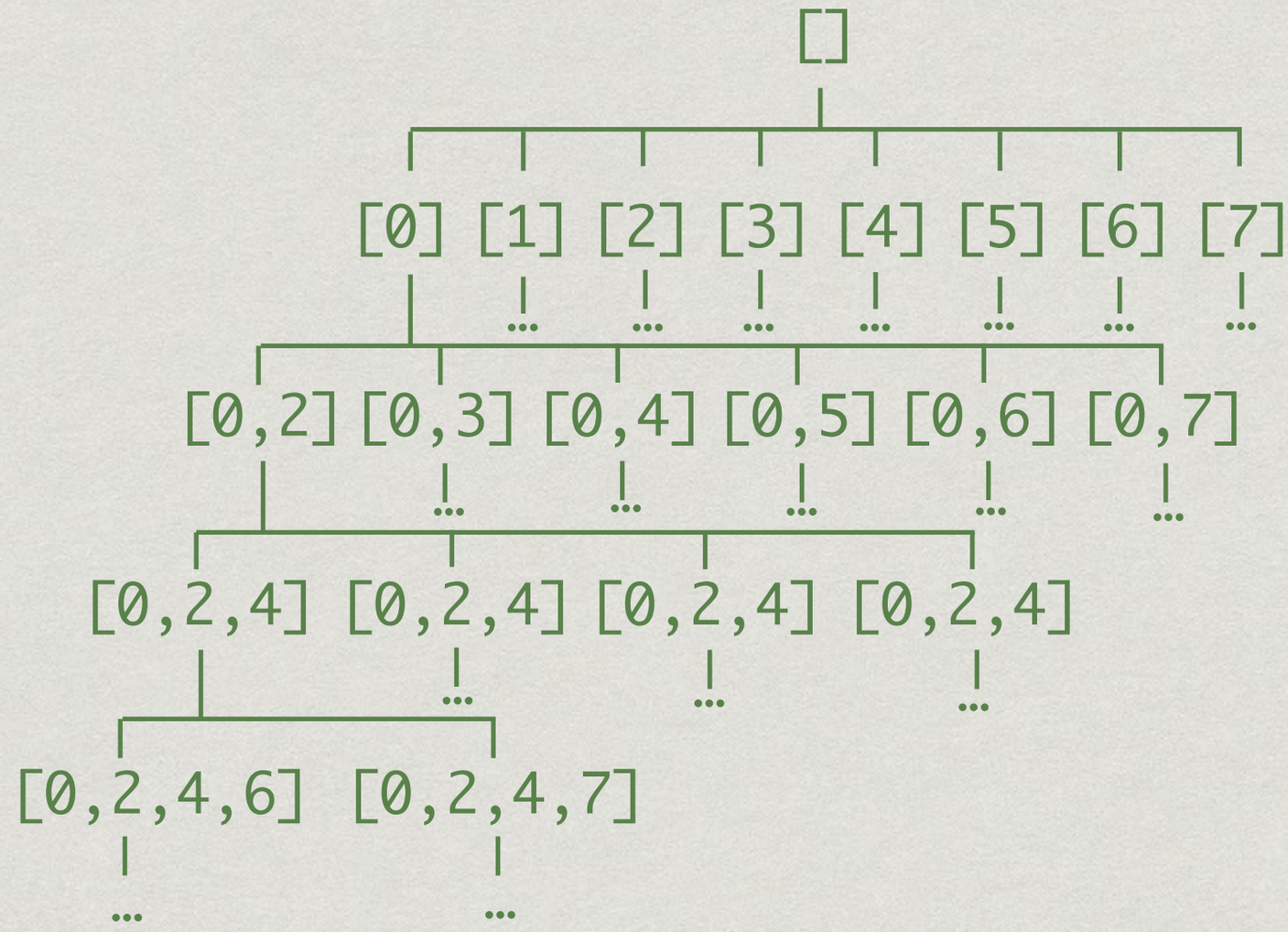
- Arrangements of k queens that have no extensions die out automatically

Laziness and efficiency



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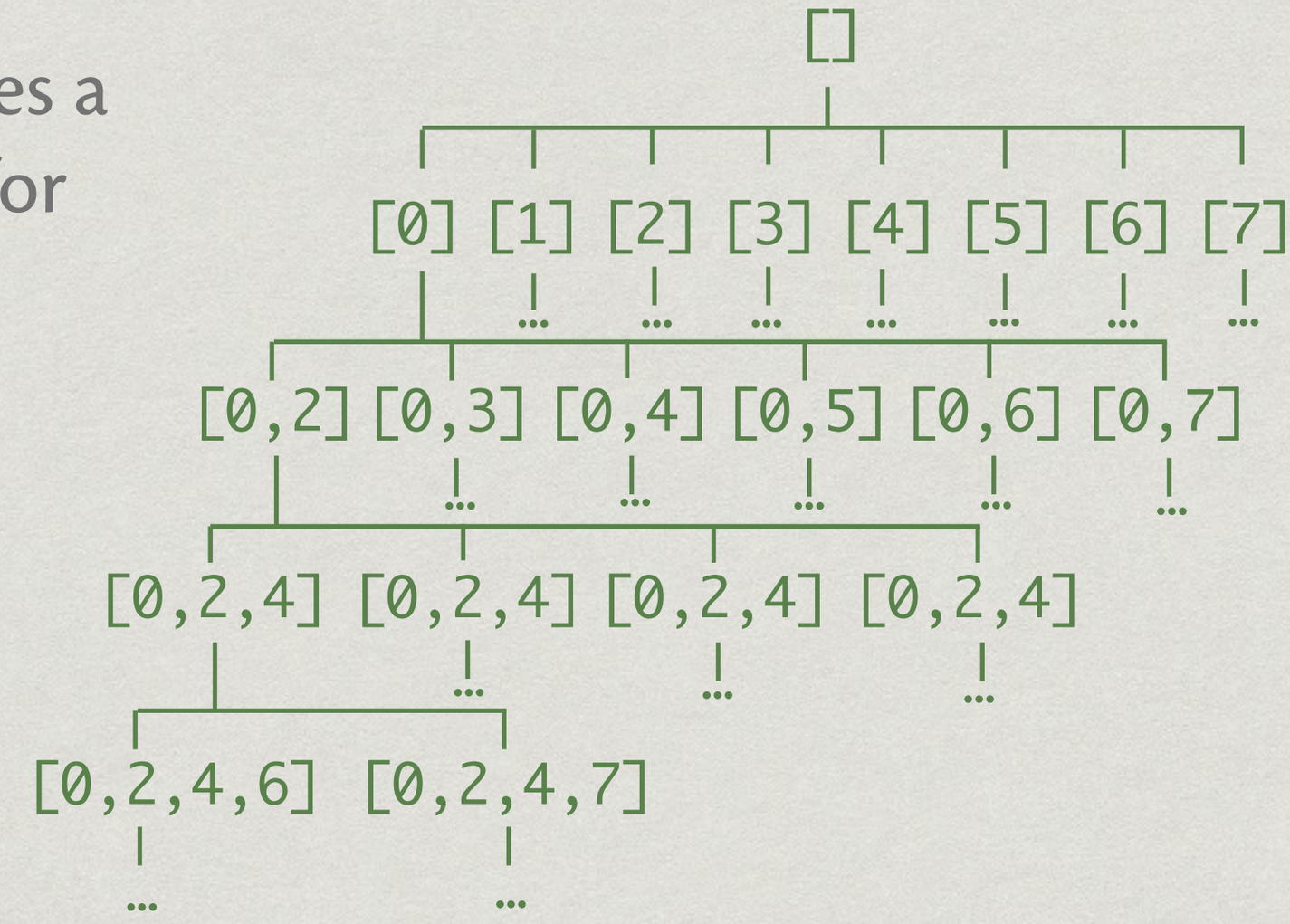
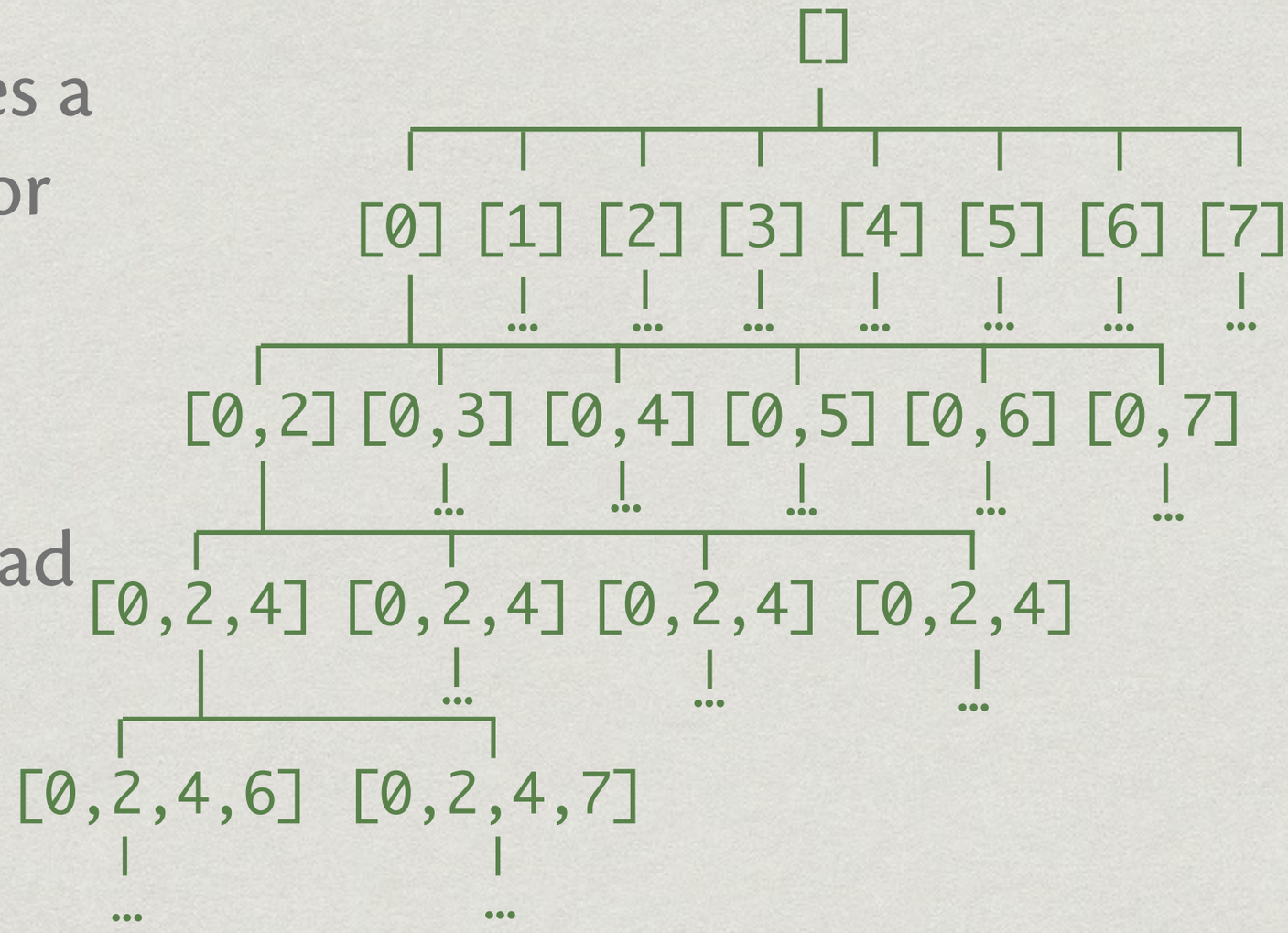


Laziness and efficiency

- Our strategy describes a breadth first search for solutions: inefficient
- Lazy evaluation of head stops with left most solution —depth first search!



Summary

- Infinite lists are a useful way to think about search problems
- Instead of backtracking, iteratively generate all valid solutions upto a desired depth
- Lazy evaluation converts inefficient breadth-first search to depth-first evaluation of leftmost solution

Polymorphism

- Functions that only look at the structure of a list work on lists of any type

```
head :: [a] -> a

length :: [a] -> Int

reverse :: [a] -> [a]

take :: Int -> [a] -> [a]
```

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- Can we sort any type of list?
- What about a list of functions
  `[factorial, (+3), (*5)] :: [Int -> Int]`
- How do we compare `f < g` for functions?

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- The type class `Ord` contains all types whose values can be compared

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  - If `a` is in `Ord`, `quicksort` is of type `[a] -> [a]`

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- How to evaluate `elem f funclist`?

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- `f == g` implies for all `x`, `f x` terminates iff `g x` does

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- Hence, equality over functions is not computable

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- Show : values that can be displayed
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  - For example, function types `a -> b` do not belong to Show

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  - `Eq` is defined by the signature `==`, `/=`
  - …
- Later we will see how to define our own type classes and add types to a type class

Summary

- A type class is a collection of types satisfying additional properties over their values
  - Check for equality, compare magnitude …
- Additional properties are defined in terms of signatures — additional functions that must be defined on all values of the type
- Can use type classes to specify conditionally polymorphic types for functions