Programming in Haskell
Aug-Nov 2015

LECTURE 10

SEPTEMBER 3, 2015

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Chennai Mathematical Institute

Combining elements

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- What is the common pattern?

Combining elements …

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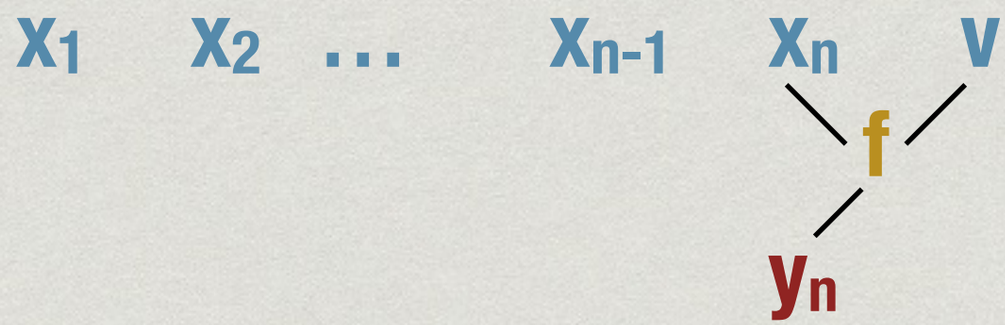

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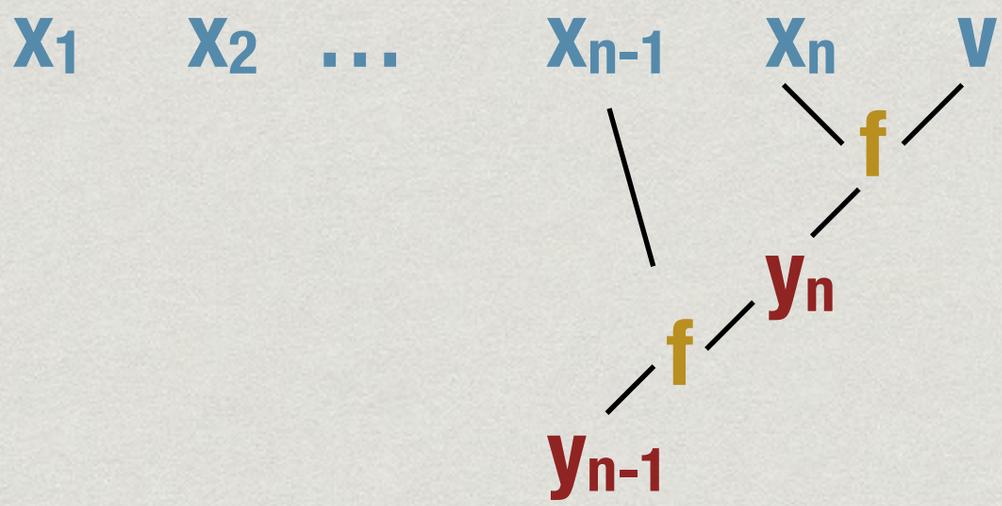

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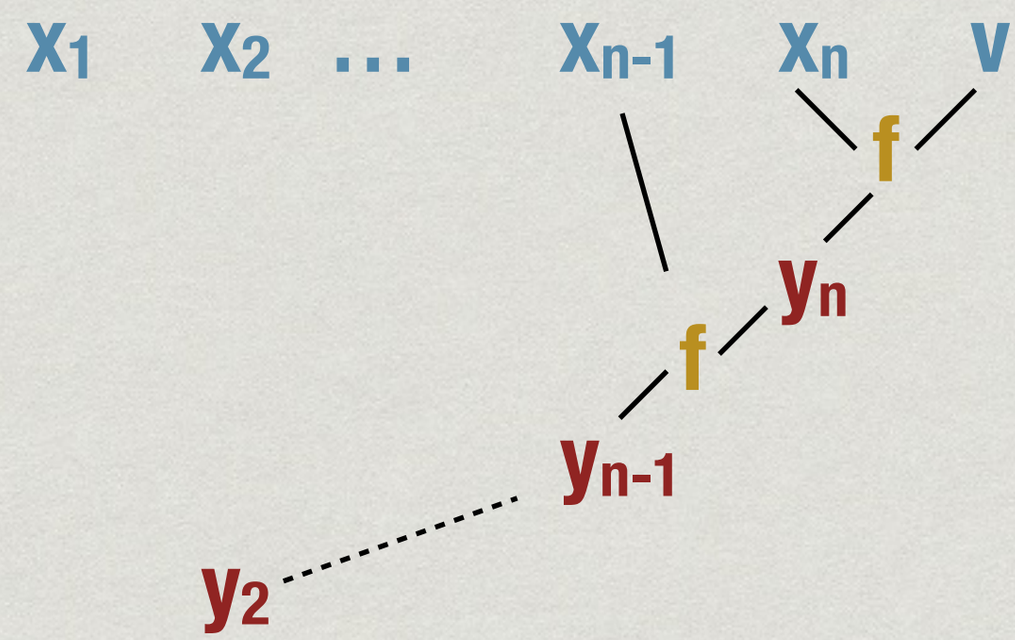

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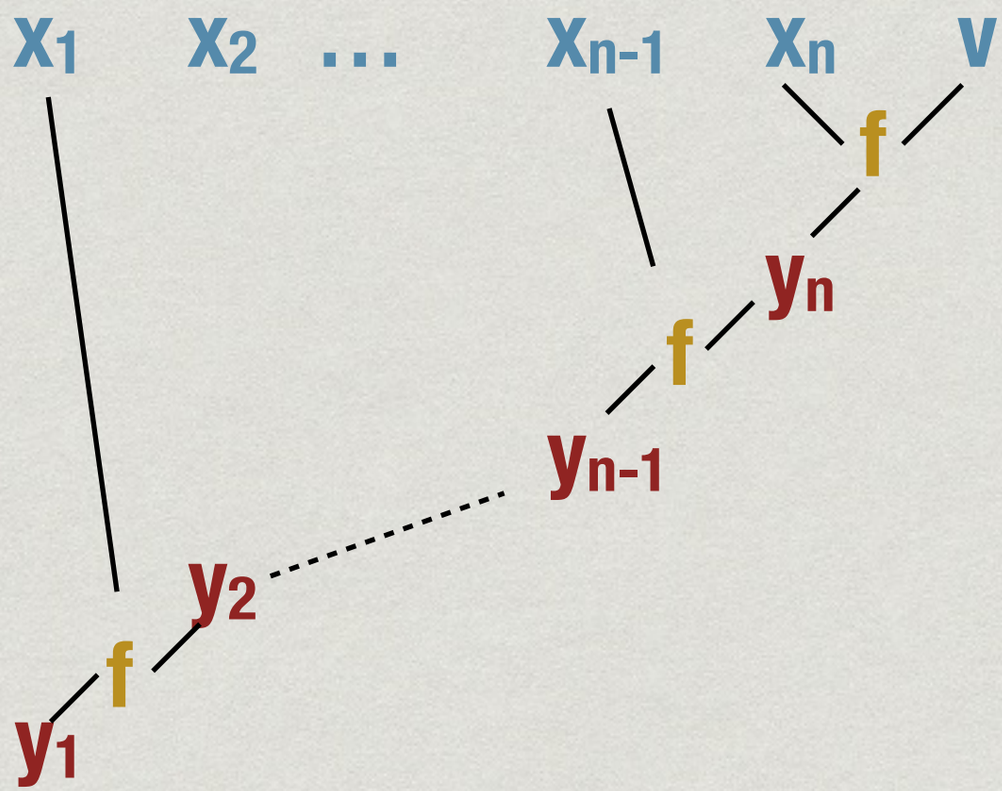

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mylength l =  foldr f 0 l
  where
  f x y = y+1
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- Note: can simply write `mylength = foldr f 0`

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mylength :: [Int] -> Int
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  where
  f x y = y+1
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- Note: can simply write `mylength = foldr f 0`
  - Outermost reduction: `mylength l ⇛ foldr f 0 l`

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```
appendright x l = l ++ [x]
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- `foldr appendright [] = ??`

Examples ...

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- `foldr appendright [] = ??`
- `foldr appendright [] = reverse`

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- The built-in function `concat`

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```
foldr1 f [x] = x
foldr1 f (x:xs) = f x (foldr1 f xs)

maxlist = foldr1 max
```

Folding from the left

- Sometimes useful to fold left to right

- ```
  foldl :: (a -> b -> a) -> a -> [b] -> a
  foldl f v [] = v
  foldl f v (x:xs) = foldl f (f v x) xs
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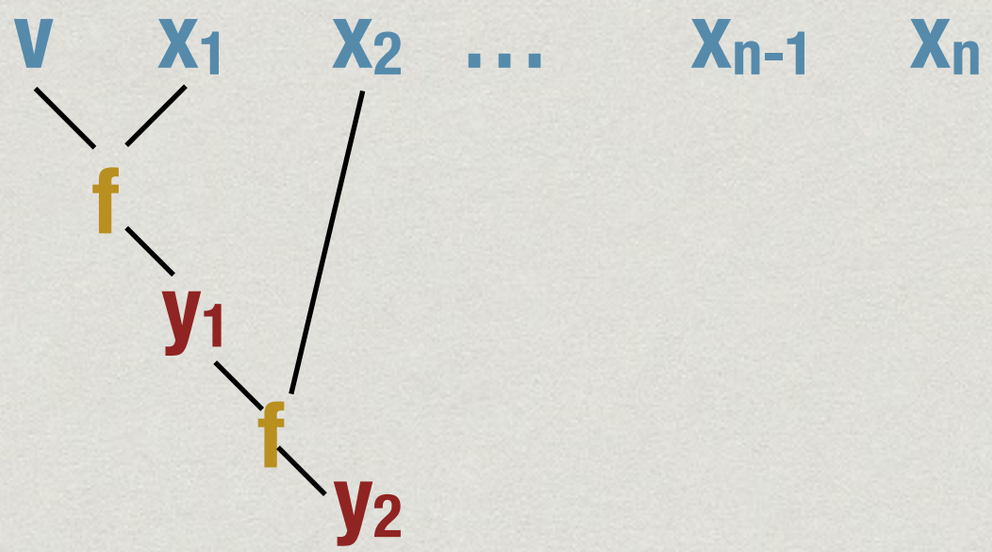
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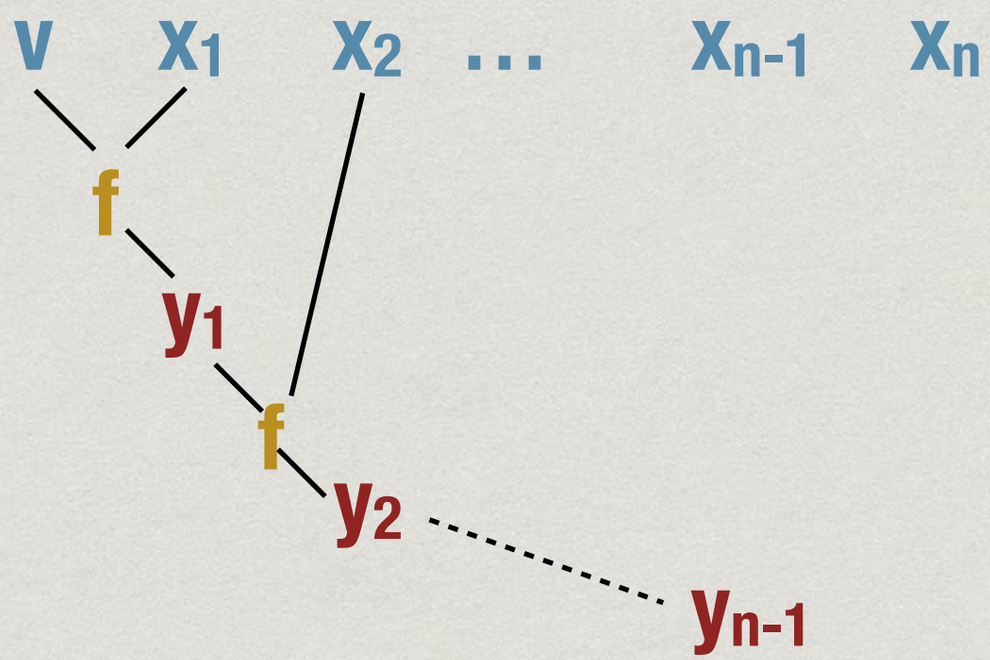
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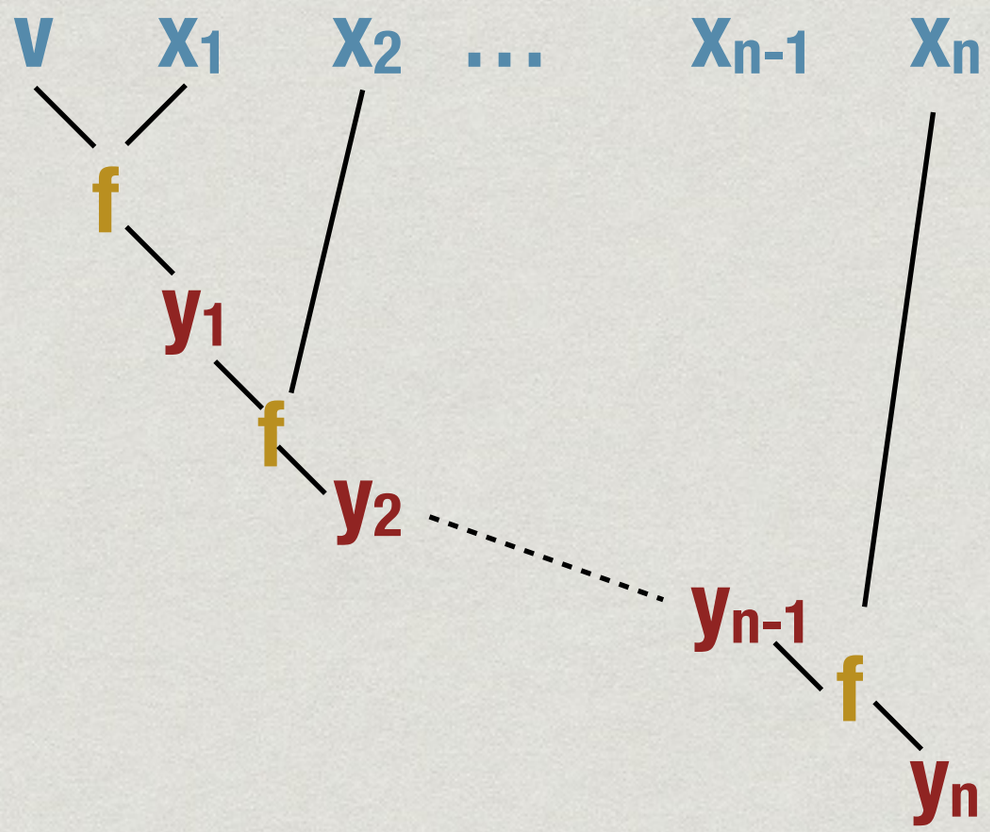
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- Convert a character into the corresponding digit:

```
chartonum :: Char -> Int
chartonum c
    | ('0' <= c) && (c <= '9')
                  = (ord c) - (ord '0')
```

Example …

- Process the digits left to right
- Multiply current sum by 10 and add next digit

```
nextdigit :: Int -> Char -> Int
nextdigit i c = 10*i + (chartonum c)

strtonum = foldl nextdigit 0
```

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- Symmetric function `dropWhile`

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  - `zipWith :: (a -> b -> c)->[a]->[b]->[c]`

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- Entry j is the mark obtained by student j
- Compute the total marks obtained by each student.

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addMarks = foldr1 (zipWith (+))
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  nondecreasing [] = True
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                        (x <= y) && (nondecreasing (y:ys))
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- Pair up `l` with `tail l` and compare position by position

- ```
  nondecreasing xs =
              and [x <= y | (x,y) <- zip xs (tail xs) ]
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```
allpos c s =
    [i | (y,i) <- zip s [0..(length s)], c == y]
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position c s = head ((allpos c s) ++ [length s])
```

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initsegs :: [a] -> [[a]]
initsegs [] = [[]]
initsegs (x:xs) =
                  []:map (x:) (initsegs xs)
```

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```
interleave :: a -> [a] -> [[a]]
interleave x [] = [[x]]
interleave x (y:ys) =
    (x:y:ys) : map (y:) (interleave x ys)
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perm [x] = [[x]]
perm (x:xs) = concat (map (interleave x) (perm xs))
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```
concatMap f l = concat (map f l)
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  part [x] = [[x]]
  part (x:xs) =
              [(x:head l):(tail l)  | l <- part xs]
            ++ [[x]:l | l <- part xs]
  ```
  ```

Summary

- Many cumulative operations on lists can be expressed in terms of folding a function through the list
- Built in functions `foldr, foldr1, foldl`
- Other higher order functions on lists:
  - `takeWhile`
  - `zipWith`
  - `zip`
- Several examples of functions on lists

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- To learn a language, we must practice speaking it