Programming in Haskell
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LECTURE 2

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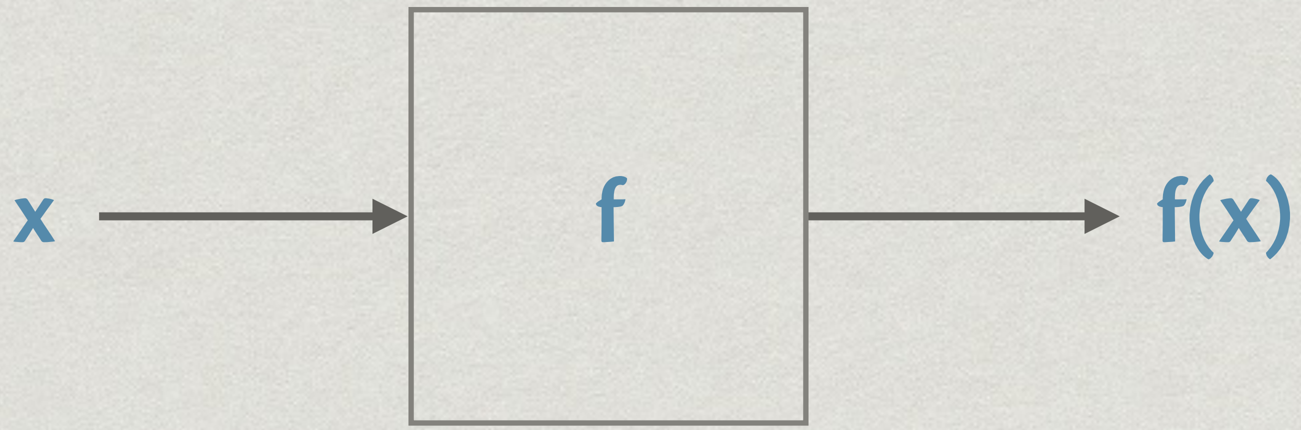
Programs as functions

- Functions transform inputs to outputs



- **Program**: rules to produce output from input
- **Computation**: process of applying the rules

Building up programs

How do we describe the rules?

- Start with built in functions
- Use these to build more complex functions

Building up programs …

Suppose

- … we have the whole numbers, `{0,1,2,…}`
- … and the successor function, `succ`

```
succ 0 = 1
succ 1 = 2
succ 2 = 3
…
```

- Note: we that write `succ 0`, not `succ(0)`

Building up programs …

We can compose `succ` twice to build a new function

- `plusTwo n = succ (succ n)`

If we compose `plusTwo` and `succ` we get

- `plusThree n = succ (plusTwo n)`

Inductive/recursive definitions

- `plus n 0 = n`, for every `n`
- `plus n 1` = `succ n = succ (plus n 0)`
- Assume we know how to compute `plus n m`
- Then, `plus n (succ m)` is `succ (plus n m)`

Computation

- Unravel the definition

- ```
  plus 7 3
  = plus 7 (succ 2)
  = succ (plus 7 2)
  = succ (plus 7 (succ 1))
  = succ (succ (plus 7 1))
  = succ (succ (plus 7 (succ 0)))
  = succ (succ (succ (plus 7 0)))
  = succ (succ (succ 7))
  ```
  ```

Inductive/recursive definitions

- `plus n 0 = n`, for every `n`
- `plus n 1` = `succ n = succ (plus n 0)`
- Assume we know how to compute `plus n m`
- Then, `plus n (succ m)` is `succ (plus n m)`

Recursive definitions …

Multiplication is repeated addition

- `mult n m` means apply `plus n`, `m` times
- `mult n 0 = 0`, for every `n`
- `mult n (succ m) = plus n (mult n m)`

Summary

- Functional programs are rules describing how outputs are derived from inputs
- Basic operation is function composition
- Recursive definitions allow repeated function composition, depending on the input

Types

Functions work on values of a fixed type

- `succ` takes a whole number as input and produces a whole number as output
- `plus` and `mult` take two whole numbers as input and produce a whole number as output
  - Can also define analogous functions for real numbers

Types

How about `sqrt`, the square root function?

- Even if the input is a whole number, the output need not be—may have a fractional part
- Number with fractional values are a different type from whole numbers
  - In Mathematics, whole numbers are often treated as a subset of fractional or real numbers

Types

Other types

* `capitalize 'a' = 'A'`,
  `capitalize 'b' = 'B'`, …
* Inputs and outputs are letters or “characters”

Functions and types

- We will be careful to ensure that any function we define has a well defined type
  - The function `plus` that adds two whole numbers will be different from another function `plus` that adds two fractional numbers

Functions have types

- A function that takes inputs of type A and produces output of type B has a type A → B
  - In Mathematics, we write f: S → T for a function with domain S and codomain T
  - A type is a just a set of permissible values, so this is equivalent to providing the type of f

Collections

- It is often convenient to deal with collections of values of a given type
  - A list of integers
  - A sequence of characters — words or strings
  - Pairs of numbers
- Such collections are also types of values

Summary

- Functions manipulate values
- Each input and output value comes from a well defined set of possible values — a type
- We will only allow functions whose type can be defined
  - Functions themselves inherit a type
- Collections of values also types

Haskell

- A programming language for describing functions
- A function description has two parts
  - Type of inputs and outputs
  - Rule for computing outputs from inputs
- Example

```
sqr :: Int -> Int      Type definition
sqr x = x * x          Computation rule
```

Basic types

- `Int`, Integers
  - Operations: `+`, `-`, `*`, `/` (Note: `/` produces `Float`)
  - Functions: `div`, `mod`
- `Float`, Floating point ("real numbers")
- `Char`, Characters, `'a'`, `'%'`, `'7'`, …
- `Bool`, Booleans, `True` and `False`

Basic types …

- `Bool`, `Booleans`, `True` and `False`
- Boolean expressions
  - Operations: `&&`, `||`, `not`
  - Relational operators to compare `Int`, `Float`, …
    - `==`, `/=`, `<`, `<=`, `>`, `>=`

Defining functions

* `xor` (Exclusive or)
  * Input two values of type `Bool`
  * Check that exactly one of them is `True`

```
xor :: Bool -> Bool -> Bool
xor b1 b2 = (b1 && (not b2)) ||
            ((not b1) && b2)
```

(why?)

Pattern matching

- Multiple definitions, by cases

```
xor :: Bool -> Bool -> Bool
xor True  False = True
xor False True  = True
xor b1    b2    = False
```

- Use first definition that matches, top to bottom
  - `xor False True` matches second definition
  - `xor True True` matches third definition

Pattern matching …

* When does a function call match a definition?
    * If the argument in the definition is a constant, the value supplied in the function call must be the same constant
    * If the argument in the definition is a variable, any value supplied in the function call matches, and is substituted for the variable (the "usual" case)

Pattern matching ...

- Can mix constants and variables in a definition

```
or :: Bool -> Bool -> Bool
or True  b    = True
or b     True = True
or b1    b2   = False
```

- `or True False` matches first definition
- `or False True` matches second definition
- `or False False` matches third definition

Pattern matching ...

- Another example

```
and :: Bool -> Bool -> Bool
and True  b = b
and False b = False
```

- The second argument is used differently in the two definitions
  - First definition: the value b determines the answer
  - Second definition: the value b is ignored

Pattern matching ...

- Another example

```
and :: Bool -> Bool -> Bool
and True  b = b
and False _ = False
```

- Symbol _ denotes a “don’t care” argument
  - Any value matches this pattern
  - The value is not captured, cannot be reused

Pattern matching ...

```
or :: Bool -> Bool -> Bool
or True _    = True
or _    True = True
or _    _    = False
```

- Can have more than one _ in a definition

Recursive definitions

- Base case: $f(0)$
- Inductive step: $f(n)$ defined in terms of smaller values, $f(n-1)$, $f(n-2)$, …, $f(0)$
- Example: factorial
  - $0! = 1$
  - $n! = n \times (n-1)!$

Recursive definitions ...

- In Haskell

```
factorial :: Int -> Int
factorial 0 = 1
factorial n = n * (factorial (n-1))
```

- Note the bracketing in `factorial (n-1)`
  - `factorial n-1` would be read as `(factorial n) - 1`
- No guarantee of termination: what is `factorial (-1)`

Conditional definitions

* Use conditional expressions to selectively enable a definition
* For instance, "fix" `factorial` for negative inputs

```
factorial :: Int -> Int
factorial 0 = 1
factorial n
  | n < 0 = factorial (-n)
  | n > 0 = n * (factorial (n-1))
```

Conditional definitions ..

```
factorial :: Int -> Int
factorial 0 = 1
factorial n
  | n < 0 = factorial (-n)
  | n > 0 = n * (factorial (n-1))
```

- Second definition has two parts
  - Each part is guarded by conditional expression
  - Test guards top to bottom
  - Note the indentation

Conditional definitions ..

```
factorial :: Int -> Int
factorial 0 = 1
factorial n
  | n < 0 = factorial (-n)
  | n > 0 = n * (factorial (n-1))
```

- Multiple definitions can have different forms
  - Pattern matching for `factorial 0`
  - Conditional definition for `factorial n`

Conditional definitions ...

- Guards may overlap

```
factorial :: Int -> Int
factorial 0 = 1
factorial n
  | n < 0 = factorial (-n)
  | n > 1 = n * (factorial (n-1))
  | n > 0 = n * (factorial (n-1))
```

Conditional definitions ...

- Guards may not cover all cases

```
factorial :: Int -> Int
factorial 0 = 1
factorial n
  | n < 0 = factorial (-n)
  | n > 1 = n * (factorial (n-1))
```

- No match for `factorial 1`

```
Program error: pattern match failure: factorial 1
```

Conditional definitions ...

- Replace the last guard by `otherwise`

```
factorial :: Int -> Int
factorial n
  | n == 0 = 1
  | n > 0  = n * (factorial (n-1))
  | otherwise = factorial (-n)
```

- "Catch all" condition, always true
- Ensures that at least one definition matches

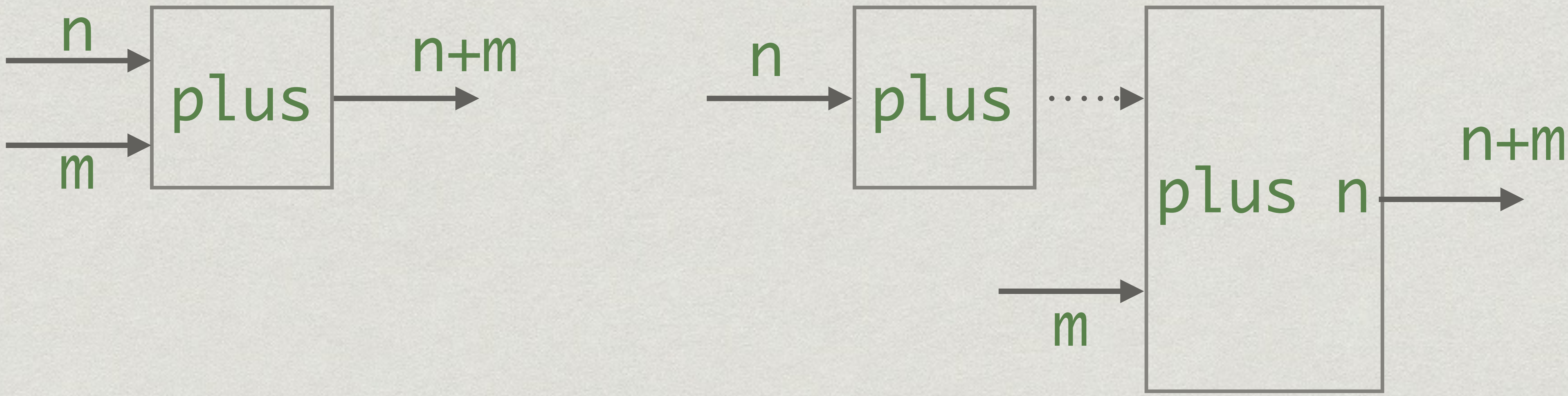
Functions with multiple inputs

Recall that we write plus n m, not plus(n,m)

- Normally, functions come with an arity
  - Number of arguments
- Instead, assume all functions take only one input!
  - This is called currying, for the logician Haskell Curry (after whom the language is also named)

Multiple inputs …

plus(n,m) = n + m   plus n m = n + m

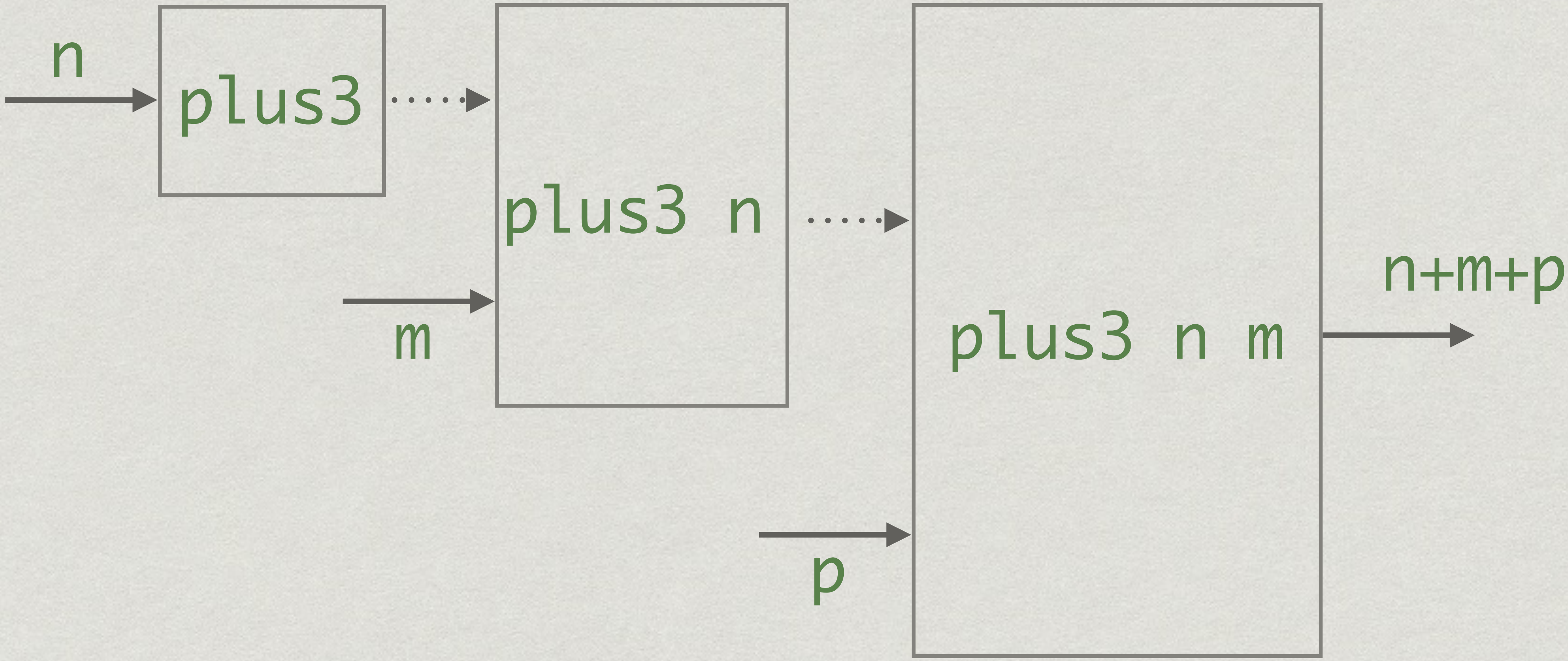


Type of plus

- plus n: input Int, output Int, so Int->Int
- plus : input Int, output Int->Int, so Int->(Int->Int)

Multiple inputs …

```
plus3 n m p = n + m + p
```




```
plus3 : Int -> (Int -> (Int -> Int))
```

Multiple inputs …

- Consider a function with many arguments

  f x1 x2 … xn = y

- Suppose each `xi` is of type `Int`, `y` is of type `Bool`
- Type of `f` is

  f :: Int -> (Int -> (… (Int->Bool)…)

- Correspondingly, we should write

  (…((f x1) x2) …) xn = y

Multiple inputs …

- Fortunately, Haskell knows this!
- Implicit bracketing for types is from the right, so

  f :: Int -> Int -> … -> Int -> Bool

  means

  f :: Int -> (Int -> (… ->(Int -> Bool)…)

Multiple inputs ...

- Likewise, function application brackets from left
- So

  f x1 x2 … xn

  means

  (…((f x1) x2) …) xn
- Which is why we have to be careful to write factorial (n-1) because, factorial n-1 means (factorial n) -1

Running Haskell programs

- Haskell interpreter ghci
  - Interactively call builtin functions
  - Load user-defined Haskell code from a text file

Setting up ghci

- Download and install the Haskell Platform
  - `https://www.haskell.org/platform/`
  - Available for Windows, Linux, MacOS

Using ghci

- Create a text file (extension .hs) with your Haskell function definitions
- Run ghci at the command prompt
- Load your Haskell code
  - `:load myfile.hs`
- Call functions interactively within ghci

Compiling

- ghc is a compiler that creates a standalone executable from a .hs file
  - ghc stands for Glasgow Haskell Compiler
  - ghci is the associated interpreter
- Using ghc requires some advanced concepts
  - We will come to this later in the course

Summary

- ghci is a user-friendly interpreter
  - Can load and interactively execute user defined functions
- ghc is a compiler
  - But we need to know more Haskell before we can use it