Predictive Analytics
Regression and Classification

Lecture 5 : Part 1

Sourish Das

Chennai Mathematical Institute

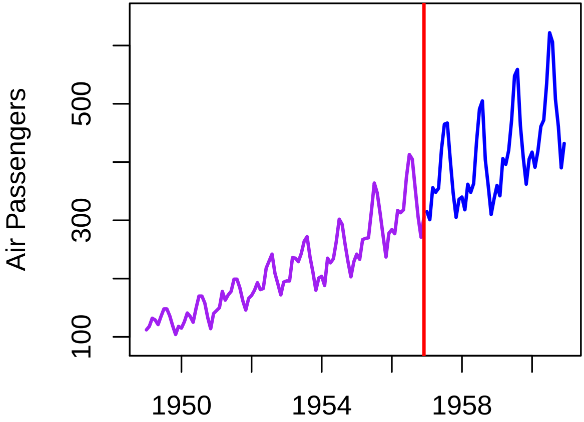
Aug-Nov, 2020

Regression Model for Time Series Data

- **Time Series** Data is ubiquitous in **Data Science**.
- **Regression Model** is one of the most commonly used statistical tools for forecasting of **time series data**.
- Two kinds of forecasting:
  1. Long term forecasting
     - modeling trend and seasonality as function of `time`
  2. Short term forecasting
     - detrend and auto-regression model

Long term forecasting with Trend

- Throughout we will consider the `AirPassengers` dataset



Long term forecasting with Trend

- Model the trend of `AirPassengers` as linear (or quadratic) function of time

- Suppose y_t is the number of Monthly totals of international airline passengers; $t = 1949 \quad to \quad 1960$.

- Linear Trend

$$y_t = \beta_0 + \beta_1 t + \epsilon_t$$

Long term forecasting with Trend

- Linear Trend

$$y_t = \beta_0 + \beta_1 t + \epsilon_t$$

```
> head(data_train)
```

	y	tm
1	112	1949.000
2	118	1949.083
3	132	1949.167
4	129	1949.250
5	121	1949.333
6	135	1949.417

Long term forecasting with Trend

- Linear Trend: $y_t = \beta_0 + \beta_1 t + \epsilon_t$

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon}$$

where $\mathbf{y} = (\mathtt{y}_1\ ,\ \mathtt{y}_2\ ,\ \cdots\ ,\ \mathtt{y}_n)^T$;

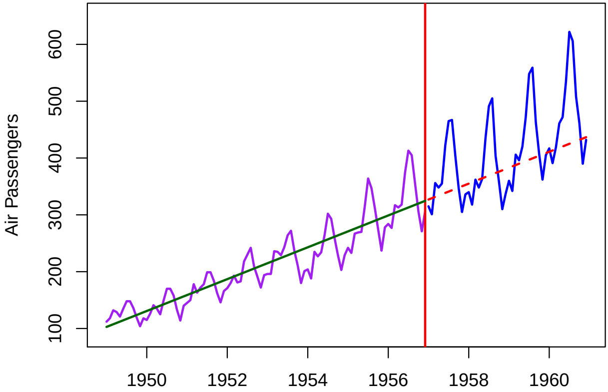
$$\mathbf{X} = \begin{pmatrix} 1 & \mathtt{t}_1 \\ 1 & \mathtt{t}_2 \\ \vdots & \vdots \\ 1 & \mathtt{t}_n \end{pmatrix}$$

$\boldsymbol{\beta} = (\beta_0, \beta_1)^T$ and $\boldsymbol{\epsilon} = (\epsilon_1, \epsilon_2, \ldots, \epsilon_n)^T$

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	213.708	3.150	67.851	0
scale(tm)	65.038	3.166	20.541	0

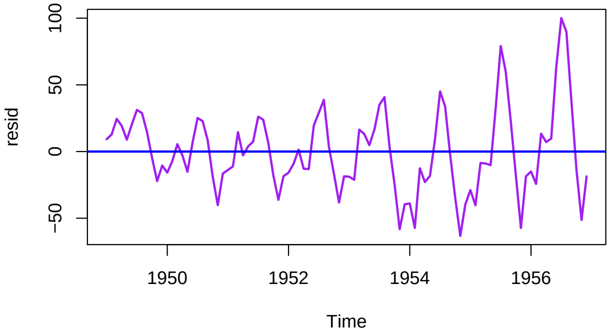
Adjusted R.Squared = 0.816

Long term forecasting with Trend



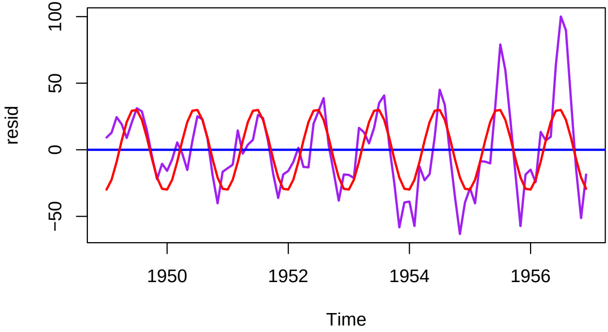
Long term forecasting with Trend

Residual shows seasonality. Now we model seasonality.



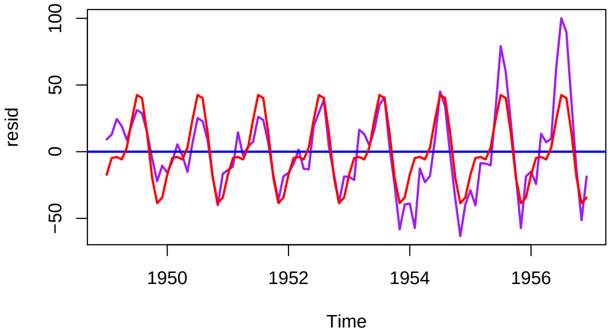
Long term forecasting with Trend

- We model seasonality as: $r_t = \alpha_0 + \alpha_1 \sin(\omega t) + \gamma_1 \cos(\omega t)$, where $\omega = \frac{2\pi}{12}$, r_t



Long term forecasting with Trend

- We model seasonality as:
  $r_t = \alpha_0 + \alpha_1 \sin(\omega t) + \gamma_1 \cos(\omega t) + \alpha_2 \sin(2\omega t) + \gamma_2 \cos(2\omega t)$,
  where $\omega = \frac{2\pi}{12}$

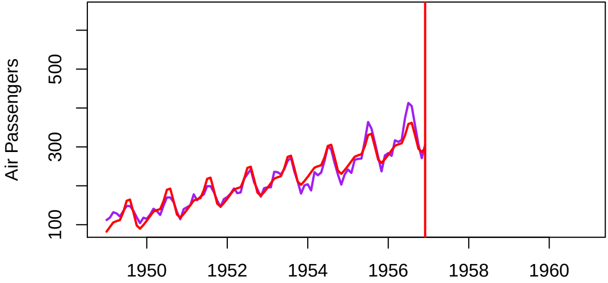


Long term forecasting with Trend & Seasonality

- We model trend & seasonality as:

$$y_t = \underbrace{\alpha + \beta t}_{trend} + \underbrace{\alpha_1 \sin(\omega t) + \gamma_1 \cos(\omega t) + \cdots}_{seasonality} + \epsilon_t,$$

where $\omega = \frac{2\pi}{12}$



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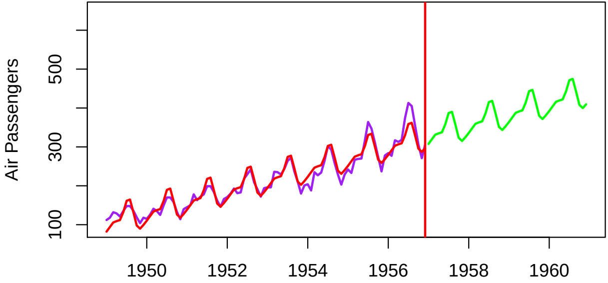

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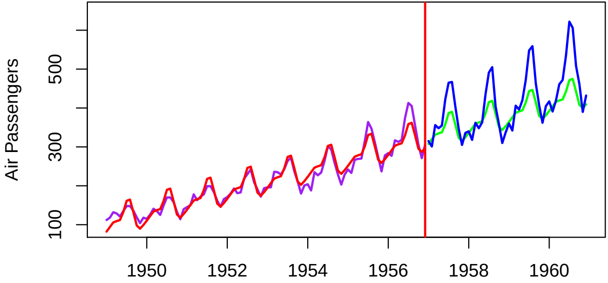
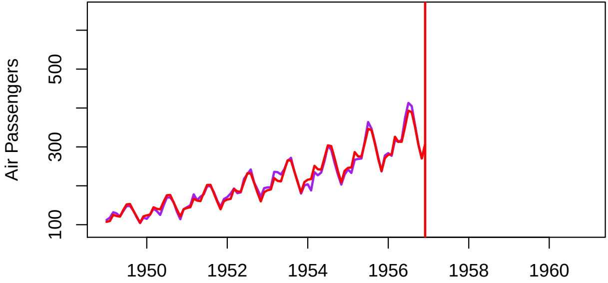


Long term forecasting with Trend & Seasonality

- Take log transformation on y:

$$\log(y_t) = \underbrace{\alpha + \beta t}_{trend} + \underbrace{\alpha_1 \sin(\omega t) + \gamma_1 \cos(\omega t) + \cdots}_{seasonality} + \epsilon_t,$$

where $\omega = \frac{2\pi}{12}$



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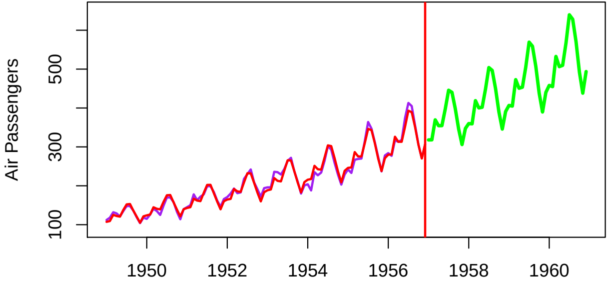

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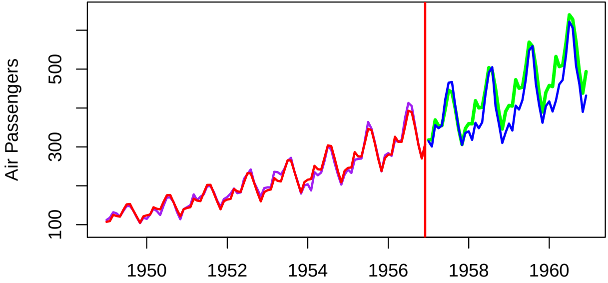


Short term forecasting: Auto-Regressive Model

- Idea of Auto-Regressive Model
- Data structure with lag-variables

$$\mathcal{D} = \begin{pmatrix} \texttt{t}_1 & y_1 & NA \\ \texttt{t}_2 & y_2 & y_1 \\ \vdots & \vdots & \vdots \\ \texttt{t}_n & y_n & y_{n-1} \end{pmatrix}$$

```
        tm   y y_lag1
1 1949.000 112     NA
2 1949.083 118    112
3 1949.167 132    118
4 1949.250 129    132
5 1949.333 121    129
6 1949.417 135    121
```

Short term forecasting: Auto-Regressive Model

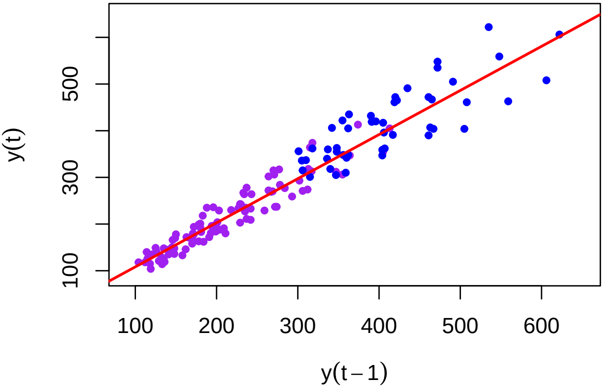
Define the first difference of $y(t)$ in log-scale

$$\delta y(t) = \ln(y(t)) - \ln(y(t-1))$$

	tm	y	y_lag1	dy	dy_lag1
1	1949.000	112	NA	NA	NA
2	1949.083	118	112	0.052	NA
3	1949.167	132	118	0.112	0.052
4	1949.250	129	132	-0.023	0.112
5	1949.333	121	129	-0.064	-0.023
6	1949.417	135	121	0.109	-0.064

Short term forecasting: Auto-Regressive Model

- Idea of Auto-Regressive Model



Short term forecasting: Auto-Regressive Model

- Auto-Regressive Model
- Model $y_t = \beta_0 + \beta_1 y_{t-1} + \epsilon_t$

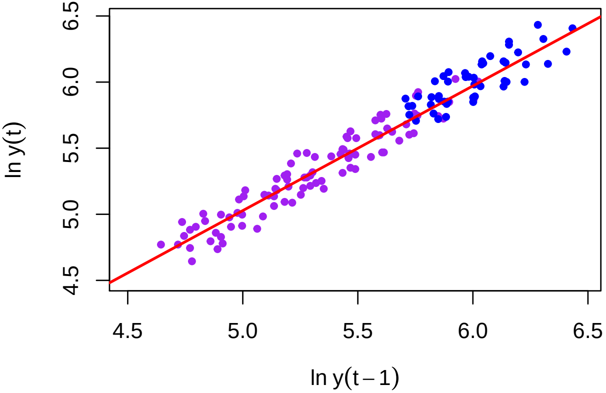
	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	13.758	7.477	1.840	0.069
y_lag1	0.945	0.033	28.354	0.000

```
        studentized Breusch-Pagan test

data:  fit_ar1
BP = 20.289, df = 1, p-value = 6.658e-06
```

Short term forecasting: Auto-Regressive Model

- Taking log transformation



Short term forecasting: Auto-Regressive Model

- Auto-Regressive Model
- Model $\log(y_t) = \beta_0 + \beta_1 \log(y_{t-1}) + \epsilon_t$

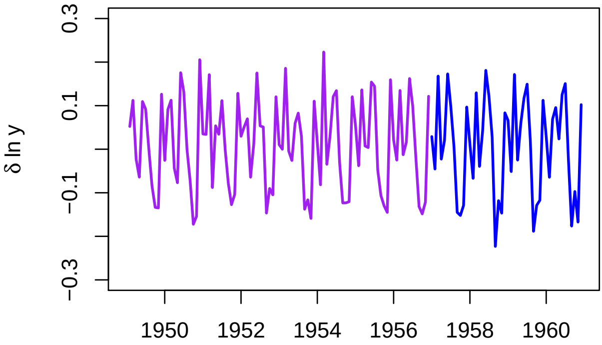
```
              Estimate Std. Error t value Pr(>|t|)
(Intercept)      0.312      0.168   1.853    0.067
log(y_lag1)      0.943      0.032  29.770    0.000
```

```
        studentized Breusch-Pagan test

data:  fit_ar1
BP = 0.051569, df = 1, p-value = 0.8204
```

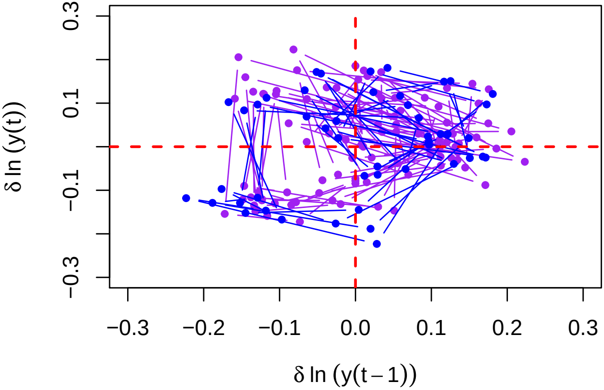
Short term forecasting: Auto-Regressive Model

- Idea of Auto-Regressive Model

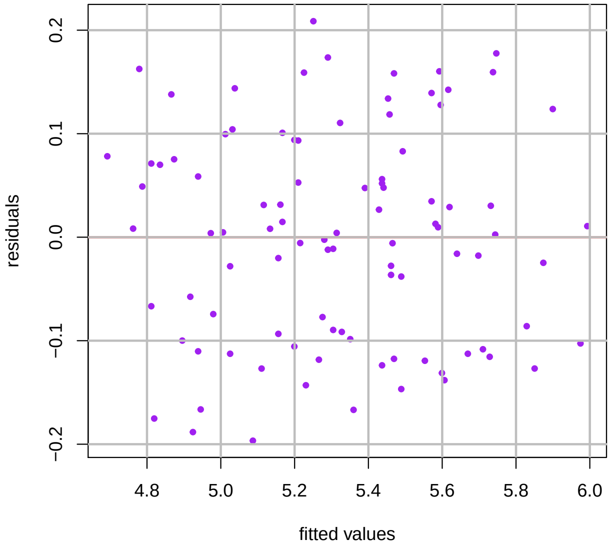


Short term forecasting: Auto-Regressive Model

- Circular topology: **Topological Data Analysis**



Short term forecasting: Auto-Regressive Model



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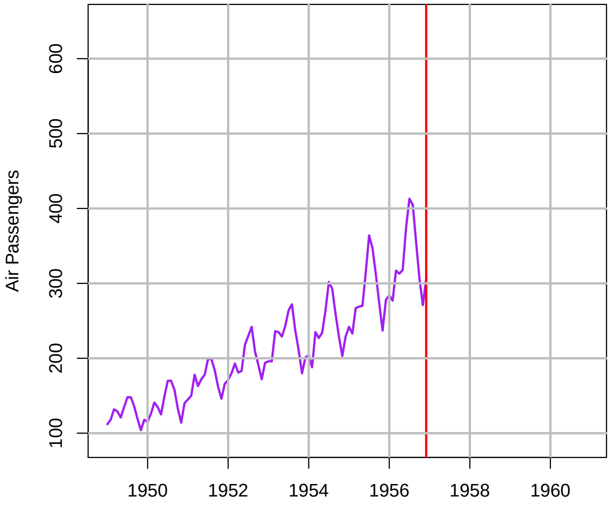

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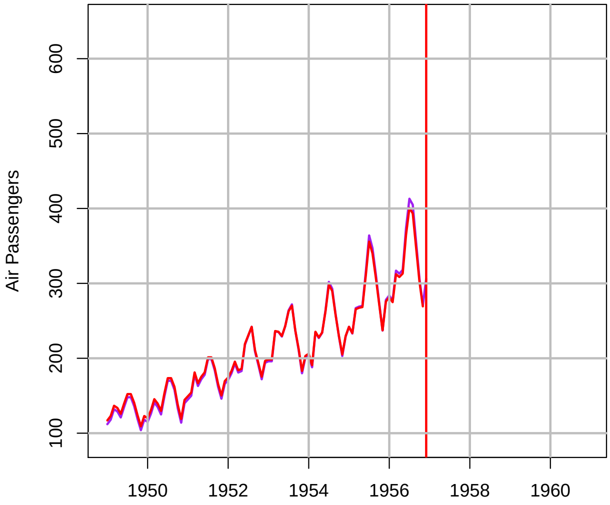
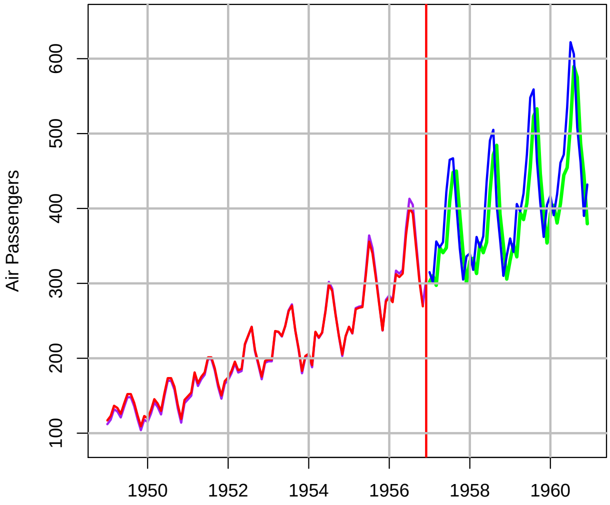


Short term forecasting: Auto-Regressive Model



Other Time Series Models

- ARIMA or GARCH model
- State-Space Dynamic model
- Spectral Analysis model etc.
- Most of these models are non-linear in parameter. Hence we cannot model them easily using regression model setup!!

In the next part ...

- Next we will discuss another nice application of Regression mode, known as the Granger Causality model

Thank You

sourish@cmi.ac.in