Research statement

Sonakshi Sachdev
Physics Department, Chennai Mathematical Institute, SIPCOT IT Park, Siruseri 603103, India
Email: sonakshi@cmi.ac.in

February 6, 2020

Ideal fluid dynamics, magnetohydrodynamics and 2-fluid plasmas have been fruitful areas of investigation relevant to aerodynamics, astrophysics, fusion plasmas, weather prediction, climate change, turbulence etc. It is well known that ideal neutral and charged flows can develop singular structures like shocks, vortex and current sheets associated for instance, with discontinuities in density or growth of enstrophy due to vortex stretching. Moreover, numerical simulations of these ideal equations often encounter finite time blow-ups. These singularities may be regularized either by dissipative or conservative mechanisms typified by viscosity as in the Navier Stokes regularization of the 3D Euler equation or by dispersion as in the KdV regularization of the 1D Hopf equation.

In [1, 2], we have developed a minimal, local, conservative, nonlinear dispersive regularization of 3D compressible flow and plasma dynamics (MHD and two-fluid). To prevent vortical singularities, we introduce quadratically nonlinear vortical and magnetic 'twirl' terms $\lambda_l^2(\mathbf{w}_l + q_l\mathbf{B}/m_l) \times (\nabla \times \mathbf{w}_l)$ in the velocity equation for each species l = ion, electron. The cut-off lengths λ_l must vary inversely with the square-roots of number densities and can be taken of order the collisionless skin depths or Debye lengths. λ_l are like position-dependent mean free paths and set the length scales at which vortex stretching should terminate. Our regularized systems, while not conserving energy and enstrophy separately (unlike in 2D) do allow for both of them to be bounded a priori through non-linear dispersive interactions. This is achieved using a Poisson structure preserving the symmetries of the original systems and is based on a conserved positive definite Hamiltonian that includes a vortical contribution $\lambda_l^2\rho_l\mathbf{w}_l^2$ in addition to the usual kinetic, compressional and electro-magnetic energies. In our regularization of Euler and MHD, a 'swirl' velocity field $\mathbf{v} + \lambda^2\nabla \times \mathbf{w}$ is identified, and shown to transport $\mathbf{w}/\rho$ and $\mathbf{B}/\rho$, generalizing the Kelvin-Helmholtz and Alfvén theorems. Flow and magnetic helicity in vortex/magnetic flux tubes are shown to be conserved. The steady regularized equations are used to model a rotating vortex, MHD pinch and a plane vortex sheet. Regularized systems of this nature can be applied to analytically and numerically discuss unsolved problems in statistical theories of 3D vortex and current singularities, (generalizing works of Onsager, Taylor and Edwards in 2D), geophysical flows, current filament/sheet dynamics in astrophysics as well as strongly nonlinear phenomena such as edge-localized modes in tokamaks.

We have also developed a new conservative regularization of shock-like singularities in adiabatic dynamics of a gas with polytropic exponent γ. It is an inviscid nonlinear dispersive counterpart to Navier Stokes with potential applications to systems where dissipative effects are small, as in nonlinear optics, weak shocks, superfluids or cold atomic gases. It may be viewed as a generalization of KdV that is valid in any dimension and describing the coupled dynamics of velocity, density, pressure and entropy. This is achieved by augmenting the ideal gas dynamic Hamiltonian by a positive density gradient term $\beta(\rho)(\nabla\rho)^2$ that prevents discontinuities in ρ from forming. The latter is reminiscent of the Korteweg capillarity energy. We have identified a capillarity coefficient $\beta(\rho) = \beta_*/\rho$ for constant β_*, that along with the standard Poisson brackets leads to local conservation laws for mass, momentum, entropy and energy suitable for describing gas dynamics. The regularization term in the resulting velocity equation is the gradient of the Bohm potential/Gross quantum pressure. Just like KdV, our equations admit sound waves with a leading cubic dispersion relation and traveling solitary and periodic waves (sech^2 and cnoidal for $\gamma = 2$). As with KdV, there are no steady shock-like solutions satisfying the Rankine-Hugoniot conditions. On the other hand, unlike in KdV, where a solitary wave train forms, our numerical studies in 1d for $\gamma = 2$ isentropic flow show that the gradient catastrophe is averted via the formation of pairs of solitary waves which display approximate phase shift scattering. The system is also shown to display recurrence. These numerical observations are related to an equivalence (via a Madelung transformation) between our model (in the special case of $\gamma = 2$ isentropic potential flow) and the cubic defocusing nonlinear Schrödinger equation (NLSE) with β_* playing the role of $\hbar^2$. Remarkably, this connection extends to any dimension and also to other values of γ where the cubic nonlinearity $|\psi|^2\psi$ is replaced by the power law $2\gamma - 1$, providing a gas dynamic interpretation of such generalizations of NLSE. More generally, our regularized gas dynamic model may be viewed as an extension of NLSE to include the dynamics of vorticity and non-barotropic pressure. Interestingly, the negative pressure version of our 1d isentropic regularized gas dynamic equations are equivalent to those of the continuous Heisenberg magnetic chain.

Some directions for future research

• Numerical solution of the initial value problem for regularized flow to determine the long-time behavior of spectral distributions of energy and enstrophy.

• Study linear instabilities in a conservatively regularized vortex sheet/rotating vortex, follow their growth and non-linear saturation due to the bound on enstrophy. This would involve a conservative compressible analogue of the Orr-Sommerfeld equation.

• Modeling oblique shocks and Sedov-Taylor spherical blast wave problem using our regularized gas dynamic equations.

• Investigate if our conservative regularization terms can arise from kinetic theory using a Chapman-Enskog-like expansion in Knudsen number.

References

[1] Krishnaswami G S, Sachdev S and Thyagaraja A, *Local conservative regularizations of compressible MHD and neutral flows*, Phys. Plasmas **23**, 022308 (2016), [arXiv:1602.04323].

[2] Krishnaswami G S, Sachdev S and Thyagaraja A, *Conservative regularization of compressible dissipationless two-fluid plasmas*, Phys. Plasmas **25**, 022306 (2018), [arXiv:1711.05236].

[3] Krishnaswami G S, Phatak S, Sachdev S and Thyagaraja A, *Nonlinear dispersive regularization of inviscid gas dynamics*, AIP Advances 10, 025303 (2020), [arXiv:1910.07836]