

Logic quiz 2

Chennai Mathematical Institute

December 2020

1. Prove that the converse of the following implications are not valid, by showing that there are structures where it is false (the underlined implications are the ones whose converse you have to prove false).

$$(a) \quad (\forall x(A(x) \leftrightarrow B(x))) \underline{\Rightarrow} (\exists xA(x) \leftrightarrow \exists xB(x)) \quad (1)$$

$$(b) \quad (\exists xA(x) \Rightarrow \forall xB(x)) \underline{\Rightarrow} (\forall x(A(x)A \Rightarrow B(x))) \quad (1)$$

$$(c) \quad \forall x(A(x) \Rightarrow B(x)) \underline{\Rightarrow} (\forall xA(x) \Rightarrow \forall xB(x)) \quad (1)$$

2. There are two linear orders, one of length 2^k and another of length $2^k - 1$. Prove that the spoiler has a winning strategy in the k round EF game. (5)
3. Assume that for all $i \leq k$, $\text{FO}[k]$ has at most N formulas in free variables x_1, x_2, \dots, x_{m+1} up to logical equivalence. How many formulas can be there in $\text{FO}[k+1]$ in free variables x_1, \dots, x_m (up to logical equivalence)? (2)