1) a)
$$\underbrace{((p\wedge\neg p)\vee a_1)}_{\phi_1}\wedge\underbrace{(a_3\vee\neg a_2)}_{\phi_2}\wedge\underbrace{(\neg a_1\vee a_2)}_{\phi_3}\wedge\underbrace{(\neg a_3\vee(a_4\wedge\neg a_5))}_{\phi_4}$$

To make $\phi_1 = 1$, $a_1 = 1$ [as $(p\wedge\neg p)$ is not satisfiable]

Hence

$a_2 = 1$ [to make $\phi_3 = 1$
$\neg a_1 \vee a_2 = 0 \vee 1 = 1$]

Hence

$a_3 = 1$ [to make $\phi_2 = 1$]

Hence

$(a_4 \wedge \neg a_5) = 1$ [to make $\phi_4 = 1$]

Hence

$a_4 = 1$, $a_5 = 0$

So

$a_1 = T$
$a_2 = T$
$a_3 = T$
$a_4 = T$
$a_5 = \perp$

3 1) b) $\neg(((a_1 \wedge \neg a_2) \vee a_3) \Rightarrow (a_3 \vee \neg a_4))$

[to satisfy $\neg(p \Rightarrow q)$ we have only one assignment $p = 1, q = 0$]

$\therefore (a_1 \wedge \neg a_2) \vee a_3 = 1$

and

$(a_3 \vee \neg a_4) = 0$ [this gives $a_3 = 0$, $a_4 = 1$]

So. $a_1 \wedge \neg a_2 = 1$ [as $a_3 = 0$]

$\therefore a_1 = 1$
$a_2 = 0$

So $a_1 = \top$
$a_2 = \bot$
$a_3 = \bot$
$a_4 = \top$

2) a) $(a \Rightarrow b) \vee (a \wedge \neg b)$

~~same as~~ $\cancel{(\neg a \vee b) \vee (a \wedge \neg b)}$

to falsify this
we need $a \Rightarrow b = 0$

[so $a = 1, b = 0$]

and $(a \wedge \neg b) = 0$

1

[so $a = 0, b = 1$]

hence

$a = 1$, $b = 1$ No assignment

b) $(\neg b \Rightarrow (a \Rightarrow c)) \vee (a \wedge b)$

to falsify this we need

$(\neg b \Rightarrow (a \Rightarrow c)) = 0$ and $(a \wedge b) = 0$

[so $a = 0, b = 0$]

or. $b \vee (a \Rightarrow c) = 0$

or. $b \vee (\neg a) \vee c = 0$ Both need not be 0.

[Putting $a = 0, b = 0$, we ~~get~~ have no value for c to make that 0]

$a = 1, b = 0, c = 0$

hence no assignment.

3) Not possible

Let $\phi[a,b]$ be an formula using $a, b, \neg, \oplus$ only

Note there are only 4 possible assignments say a_1, a_2, a_3, a_4

Claim. $\phi[a,b]$ will always have only even many satisfiable assignment.

Base case is $\phi[a,b] = a$

a	b	ϕ
0	1	0
0	0	0
1	0	1
1	1	1

dr. None will satisfy

Basically even no. of satisfying assignme

ii) $\phi[a,b] = \neg a$

a	b	ϕ
0	1	1
0	0	1
1	0	0
1	1	0

iii) $\phi[a,b] = a \oplus b$

follows from the truth tabl

iv) $\phi[a,b] = a \oplus a$

//

Same for. $\phi = b, \neg b, b \oplus b$

v) $\phi(a,b) = a \oplus \neg a$

allways true

inductive step

assume it is true for

on ~~Φ~~ $\phi[a,b]$

with $|\phi| < n$

Now for ϕ, with $|\phi| = n$

we say

i) $\phi = \neg\phi'$, where $|\phi'| = n-1$

The case analysis is [holds the statement as it is true for $\forall\, |\phi| \leq n-1$]

confusing.

ii) $\phi = \phi_1 \oplus \phi_2$

It has to be

~~for. if $\phi_1 = 1$~~

a little more structured.

$\therefore$ let the 4 assignments be a_1, a_2, a_3, a_4

and

$\phi_1 = 1$ for. a_1, a_2

if $\phi_2 = 1$ for. a_1, a_2

then $\phi_1 \oplus \phi_2 = 0$ for. a_1, a_2

$\phi_1 \oplus \phi_2 = 1$ for. a_3, a_4

if $\phi_2 = 1$ for. only one of a_1 or a_2

say for a_1

and for. a_3 (say

$\phi_1 \oplus \phi_2 = 0$ for. a_1, a_4

$\phi_1 \oplus \phi_2 = 1$ for. a_2, a_3

6

if $\phi_2 = 1$ for a_3, a_4

then $\phi_1 \oplus \phi_2 = 1$ for any assignment.

Hence our claim is true

Now if

$\phi[a,b] = a \wedge b$ $\quad a \Rightarrow b$

then it can not have even many satisfaction.

$\longrightarrow \leftarrow$

8

1)

$X \rightarrow$ finitely satisfiable

$\Rightarrow$ $X \rightarrow$ satisfiable [proved in the class]

$\Rightarrow$ $\exists\, v$, s.t. $\forall \phi \in X$, $v(\phi) = \top$

5

Now for α, if $v(\alpha) = T$, then $X \cup \alpha$ satisfiable

else $v(\neg\alpha) = T$,

hence $X \cup \{-\alpha\}$ satisfiable

$\Rightarrow$ one of $X \cup \{\alpha\}$ or $X \cup \{-\alpha\}$

satisfiable

5) :

Let C represents the chain of ~~all~~ finitely satisfiable ~~lean~~ set constructing from this. Let $\alpha_1, \alpha_2, \ldots$ be the ~~for to~~ formulals out side X

now $C_0 = X$

$C_i = C_{i-1} \cup \{\alpha_i\}$ or. $C_{i-1} \cup \{\neg\alpha_i\}$

which one is satisfiable (from 4)

Say $M = \bigcup C_i$

Claim M is finitely satisfiable

4

for any finite $S \subseteq M$

we have C_i s.t. $S \subseteq C_i$

why is Zorns lemma needed?

$C_i \rightarrow$ finitely satisfiable

$\Rightarrow S \rightarrow$ is satisfiable

$\Rightarrow M$ is finitely satisfiable

Hence $\exists$ an maximal chain from ~~Zorn's~~ lemma