Logic quiz 1

Chennai Mathematical Institute

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1. For each of the following, find a satifying truth aignment if one exists.

   (a) $((p \wedge \neg p) \vee a_1) \wedge (a_3 \vee \neg a_2) \wedge (\neg a_1 \vee a_2) \wedge (\neg a_3 \vee (a_4 \wedge \neg a_5))$ (2)

   (b) $\neg(((a_1 \wedge \neg a_2) \vee a_3) \Rightarrow (a_3 \vee \neg a_4))$ (1)

2. For each of the following, find a falsifying truth aignment if one exists.

   (a) $(a \Rightarrow b) \vee (a \wedge \neg b)$ (1)

   (b) $(\neg b \Rightarrow (a \Rightarrow c)) \vee (a \wedge b)$ (1)

3. The propositonal connective XOR has the following semantics.

$$a \text{ XOR } b = \begin{cases} \top & \text{if } a = \top, b = \bot \\ \top & \text{if } a = \bot, b = \top \\ \bot & \text{otherwise} \end{cases}$$

   Using only the variables a, b and connectives $\neg$, XOR, can you write a formula that is logically equivalent to $a \Rightarrow b$? Justify your answer. *Hint: look at what fraction of the possible truth assignments are set to true by formulas bult using* $a, b, \neg$, XOR. (10)

4. Suppose α is a formula and X is a finitely satisfiable set of formulas. Prove that at least one of $X \cup \{\alpha\}, X \cup \{\neg\alpha\}$ is fintely satisfiable. (5)

5. A set of formulas X is maximal if for every formula α, either $\alpha \in X$ or $\neg\alpha \in X$. Prove that every finitely satifsiable set can be extended to one that is maximal and finitely satisfiable. (5)