MIDSEM 2021

Answer as much as you can. The grading is relative

1. Show that, if $P = NP$ then every languge $A \in P$ except $A = \phi$ and $A = \Sigma^*$ is NP-complete.

2. Show that, if $P = NP$, we can factor integers in polynomial time.

3. Identify the error in the $P \neq NP$ proof:
Consider an algorithm for SAT: input ϕ, try all possible assignments to check if ϕ is satisfiable. So $SAT \notin P$
Thus SAT requires exponential time. Hence $P \neq NP$.
Since $P \neq EXP$.

4. Let MAX-CLIQUE $= \{\langle G, k\rangle \mid$ the largest clique in G has k vertices$\}$. Show that if $P = NP$, then MAX-CLIQUE is in P. Then also give a polynomial time algorithm that finds a largest clique.

5. Let $EQ_{REX} = \{\langle R, S\rangle \mid R$ and S are equivalent regular expressions$\}$.
Show that $EQ_{REX} \in PSPACE$

6. Let A be the languge of properly nested parentheses. Show that $A \in L$.
For e.g. $(()) \in A$ but $)(\notin A$.

7. Prove that an oracle C exists such that $NP^C \neq coNP^C$.

8. Consider the function $pad : \Sigma^* \times N \to \Sigma^* \#^*$ as follows. Let $pad(s, l) = s\#^j$ where $j = \max(0, l-n)$ and $n = |s|$.
For any language A and function $f : N \to N$ define the following language:

$$pad(A, f(n)) = \{ pad(s, f(n)) \mid |s| = n \}$$

Prove that if $A \in TIME(n^2)$ then $pad(A, n^2) \in TIME(n)$.

9. Describe a decidable language in $P/poly$ which is $\notin P$.

10. Show that if $P = NP$ then $\exists L$ there exists a language $L \in EXP$ such that L needs exponential size circuit.