

### Assignment 3

1. ZPP is the complexity class which contains all the languages  $L$  for which there is a machine  $M$  that runs in expected polynomial time but never makes a mistake on any input. Prove that  $ZPP = RP \cap coRP$ .
2.  $B$  reduces to  $C$  under a randomized polynomial time reduction, denoted by  $B \leq_r C$  if there is a probabilistic TM  $M$  s.t.  $\forall x: \Pr[B(M(x)) = C(x)] \geq \frac{2}{3}$ .  
 $BP \cdot NP = \{L : L \leq_r 3SAT\}$ .  
Prove that  $BP \cdot NP \subseteq NP/poly$ .
3. Prove that there exists a perfectly complete  $AM[O(1)]$  protocol for proving a lower bound on set size.
4. Let  $k \leq n$ . Construct a family  $\mathcal{H}_{n,k}$  of a pairwise independent functions:  $\{0,1\}^n \rightarrow \{0,1\}^k$  as discussed in class.
5. Prove QUADREQ is NP-complete.