Take home (Final)

1. Define a language L to be downward-self-reducible if there is a polynomial-time algorithm R that for any n and $x \in \{0,1\}^n$, $R^{L_{n-1}}(x) = L(x)$ where by L_k we denote an oracle that solves L on input of size at most k. Prove that if L is downward-self-reducible then $L \in PSPACE$.

2. Let L be the language of pairs $\langle A, k \rangle$ s.t A is 0/1 matrix and $perm(A) = k$ $(k \in \mathbb{Z})$. Prove that $L \in PCP(poly(n), poly(n))$.

3. Show that if $SAT \in PCP(r(n), 1)$ for $r(n) = o(\log n)$ then $P = NP$.

4. Express the Majority function on 3 variables $MAJ(u_1, u_2, u_3)$ in its Fourier basis.

5. A language $L \subseteq \{0,1\}^*$ is sparse if $\exists$ a polynomial p s.t $|L \cap \{0,1\}^n| \leq p(n)$ $\forall n$. Show that $L \in P/poly$. If L is NP complete, show that $P = NP$.