Temporal Difference Learning

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Based on slides by Madhavan Mukund

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Adding bootstrapping to Monte Carlo methods

- Dynamic programming: use generalized policy iteration to approximate π_*, v_*
  - Bootstrap from an initial estimate through incremental updates
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- Temporal Difference (TD) learning
  - Learn immediately from the ongoing episode.

From Monte Carlo to TD

- Monte Carlo update for non-stationary environments
  - $V(S_t) \leftarrow V(S_t) + \alpha[G_t - V(S_t)]$, $\alpha \in (0, 1]$ is a constant
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- Instead
  - Observe that, $R_{t+1} + \gamma V(S_{t+1})$ is our current estimate for G_t.
  - Revised update rule: $V(S_t) \leftarrow V(S_t) + \alpha[R_{t+1} + \gamma V(S_{t+1}) - V(S_t)]$
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- Also called TD(0), because it has zero lookahead
  - More generally, can look ahead n steps to update, TD(n)
  - Most general version is called TD(λ), we only consider TD(0)

TD(0) algorithm for policy evaluation

Tabular TD(0) for estimating v_π

Input: the policy π to be evaluated
Algorithm parameter: step size $\alpha \in (0, 1]$
Initialize $V(s)$, for all $s \in \mathcal{S}^+$, arbitrarily except that $V(terminal) = 0$

Loop for each episode:
  Initialize S
  Loop for each step of episode:
    $A \leftarrow$ action given by π for S
    Take action A, observe R, S'
    $V(S) \leftarrow V(S) + \alpha\big[R + \gamma V(S') - V(S)\big]$
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TD(0) example: Driving home from work

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State	*Elapsed Time (minutes)*	*Predicted Time to Go*	*Predicted Total Time*
leaving office, friday at 6	0	30	30
reach car, raining	5	35	40
exiting highway	20	15	35
2ndary road, behind truck	30	10	40
entering home street	40	3	43
arrive home	43	0	43

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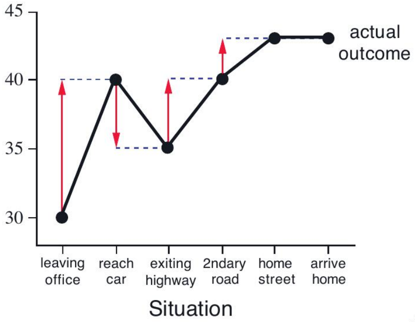
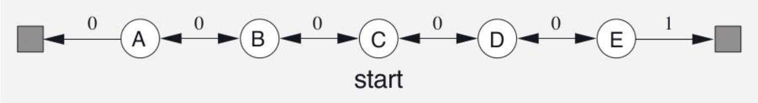


Comparing MC and TD(0)

- Markov Reward Process: MDP without actions, environment changes automatically.



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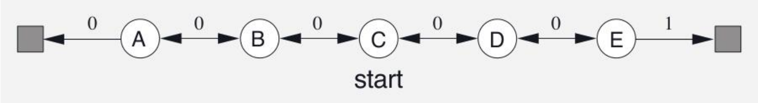

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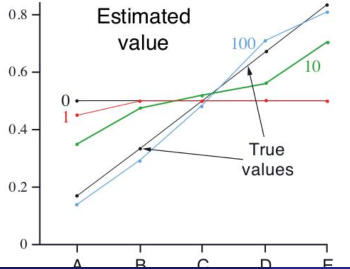

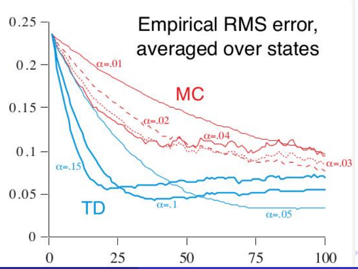

Comparing MC and TD(0) ...

Predict the values of states A and B, given the following eight episodes

A, 0, B, 0	B, 1
B, 1	B, 1
B, 1	B, 1
B, 1	B, 0

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- $V(B) = 6/8 = 0.75$

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- What about $V(A)$?

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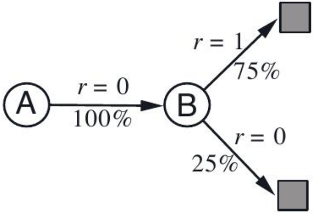
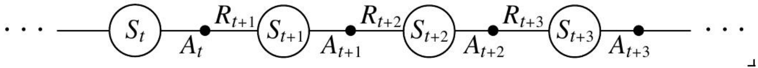
- What about $V(A)$?
- MC — only one episode with A with total reward 0, hence $V(A) = 0$
- TD(0) — $V(A) = 0.75$, because based on data, we always go from A to B with reward 0, and $V(B) = 0.75$.



SARSA: On policy TD control, estimating π_*

- For π_*, better to estimate q_π rather than v_π
- Structure of an episode



- Use the following update rule
  $Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha[R_{t+1} + \gamma Q(S_{t+1}, A_{t+1}) - Q(S_t, A_t)]$
- Update uses $(S_t, A_t, R_{t+1}, S_{t+1}, A_{t+1})$, hence the name SARSA
- As with Monte Carlo estimation, use ε-soft policies to balance exploration and exploitation

SARSA algorithm on-policy TD control

Sarsa (on-policy TD control) for estimating $Q \approx q_*$

Algorithm parameters: step size $\alpha \in (0, 1]$, small $\varepsilon > 0$
Initialize $Q(s, a)$, for all $s \in \mathcal{S}^+, a \in \mathcal{A}(s)$, arbitrarily except that $Q(\mathit{terminal}, \cdot) = 0$

Loop for each episode:
    Initialize S
    Choose A from S using policy derived from Q (e.g., ε-greedy)
    Loop for each step of episode:
        Take action A, observe R, S'
        Choose A' from S' using policy derived from Q (e.g., ε-greedy)
        $Q(S, A) \leftarrow Q(S, A) + \alpha\big[R + \gamma Q(S', A') - Q(S, A)\big]$
        $S \leftarrow S'$; $A \leftarrow A'$;
    until S is terminal

Q-learning: Off policy TD control, estimating π_*

- Directly estimate q_* independent of policy being followed
- Use the following update rule
  $Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha[R_{t+1} + \gamma \max_a Q(S_{t+1}, a) - Q(S_t, A_t)]$
- Observe that we use the *greedy policy* at S_{t+1}, unlike SARSA which uses the policy π. This comes from the *Bellman equations.*
- Underlying policy still needs to be designed to visit all state-action pairs
- With suitable assumptions, Q-learning provably converges to q_*

Q-learning algorithm, off-policy TD control

Q-learning (off-policy TD control) for estimating $\pi \approx \pi_*$

Algorithm parameters: step size $\alpha \in (0, 1]$, small $\varepsilon > 0$
Initialize $Q(s, a)$, for all $s \in \mathcal{S}^+, a \in \mathcal{A}(s)$, arbitrarily except that $Q(\mathit{terminal}, \cdot) = 0$

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Summary

- Temporal difference methods combine bootstrapping with Monte Carlo exploration of state space
- SARSA is a TD(0) algorithm for on-policy control — estimating π_*
- Q-learning is an off-policy algorithm that provably converges to q_*
- TD-based approaches apply beyond reinforcement learning
  - General methods to make long term predictions about dynamical systems
- Theoretical properties such as convergence still an area of research