Markov Decision Processes

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based on slides by Madhavan Mukund

Advanced Machine Learning 2022

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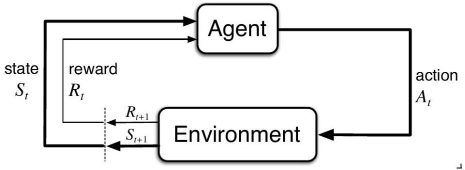
  Trajectory $S_0, A_0, R_1, S_1, A_1, R_2, S_2, \ldots$



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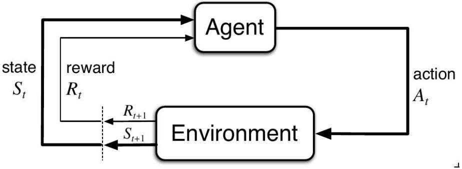

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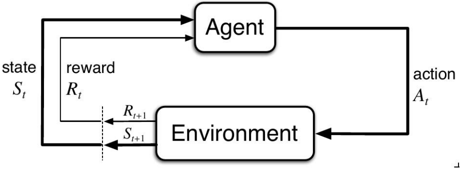

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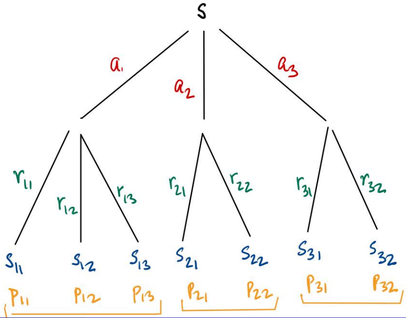

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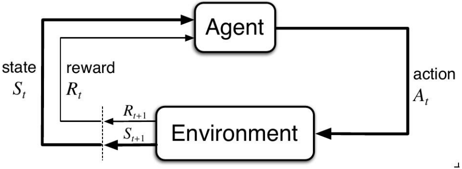
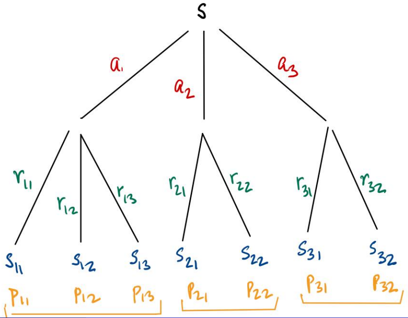
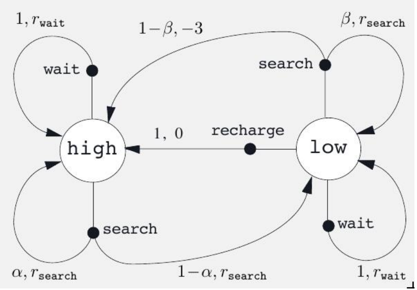
- Typically assume finite MDPs — S, A and R are finite





MDP Example: Robot that collects empty cans

- State — battery charge: high, low
- Actions: search for a can, wait for someone to bring can, recharge battery
  - No recharge when high



s	a	s'	$p(s' \mid s, a)$	$r(s, a, s')$
high	search	high	α	r_{search}
high	search	low	$1-\alpha$	r_{search}
low	search	high	$1-\beta$	-3
low	search	low	β	r_{search}
high	wait	high	1	r_{wait}
high	wait	low	0	-
low	wait	high	0	-
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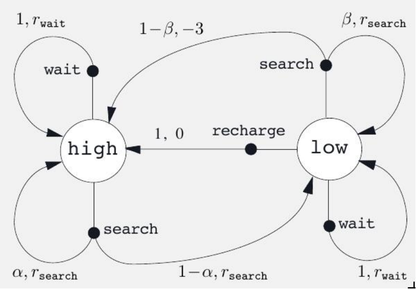

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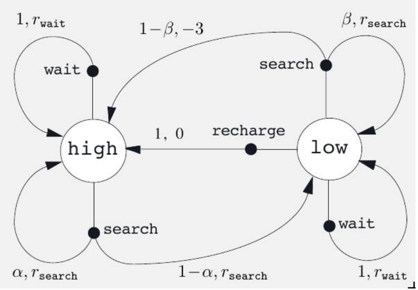
- 1 unit of reward per can collected
- $r_{search} > r_{wait}$ — cans collected while searching, waiting
- Negative reward for requiring rescue (low to high while searching)



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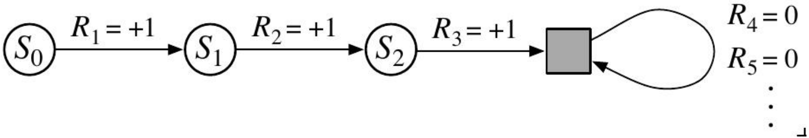

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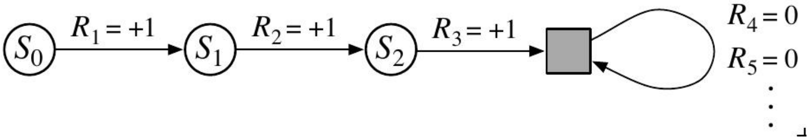

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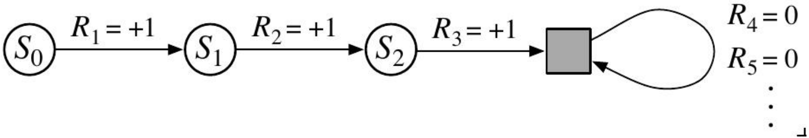

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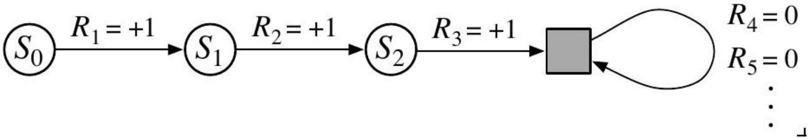

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- If $T = \infty$, $R_k = +1$ for each k, $\gamma < 1$, then $G_t = \frac{1}{1-\gamma}$

Policies and value functions

- A policy π describes how the agent chooses actions at a state
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$$v_\pi(s) \triangleq \mathbb{E}_\pi[G_t \mid S_t = s] = \mathbb{E}_\pi\left[\sum_{k=0}^{\infty} \gamma^k R_{t+k+1} \mid S_t = s\right]$$

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- Goal is to find an optimal policy, that maximizes state/action value at every state

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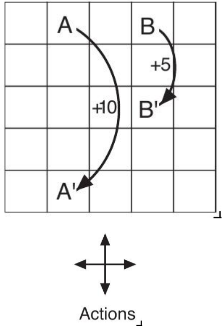
- Bellman equation relates state value at s to state values at successors of s
- Value function v_π is unique solution to the equation

Gridworld Example

- Actions in each cell are {N,S,E,W}, with usual interpretation



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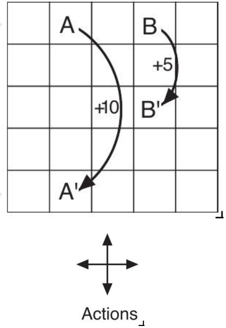

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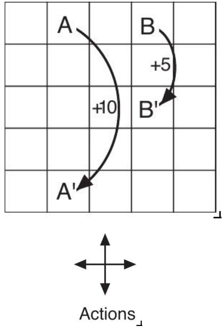

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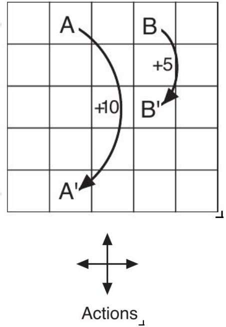

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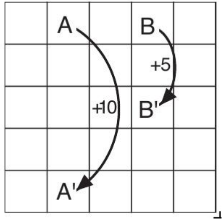

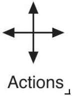

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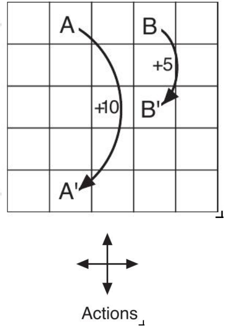

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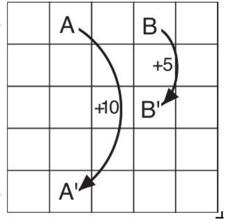

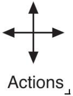

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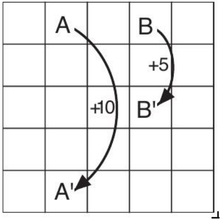
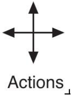
- Value at B is more than 5 because next move is to a square with positive value





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- Instead, we will explore iterative methods to approximate v_*