<u>Notation</u>: In an abelian category $\mathcal{A}$, if

$$\varphi : \bigoplus_{j=1}^{n} M_j \longrightarrow \bigoplus_{i=1}^{m} N_i$$

is a morphism, the direct sums being finite direct sums, then φ can be written as

$$\varphi = \begin{pmatrix} \varphi_{11} & \cdots & \varphi_{1n} \\ \vdots & & \vdots \\ \varphi_{m1} & \cdots & \varphi_{mn} \end{pmatrix}$$

with

$$\varphi_{ij} : M_j \longrightarrow N_i$$

the component map under the decomposition

$$\mathrm{Hom}_{\mathcal{A}}\Big(\bigoplus_j M_j, \bigoplus_i N_i\Big) = \prod_{i,j} \mathrm{Hom}(M_j, N_i).$$

The advantage of this notation is that the composition of maps

$$\bigoplus_k L_k \xrightarrow{\psi} \bigoplus_j M_j \xrightarrow{\varphi} \bigoplus_i N_i$$

can be written as the matrix product $(\varphi_{ij})(\psi_{jk})$.

Using this Problem 1 of HW-II has the following soln.

$$\tilde{T}^n = K^{n+1} \oplus T^n = C^n_{\varphi}.$$

Define ψ^n by

$$\psi^n = \begin{pmatrix} (-1)^{n-m+1} & 0 \\ 0 & 1 \end{pmatrix}$$

We have (by defn)

$$\partial^n_{\tilde{T}} = \begin{pmatrix} \partial^{n+1}_K & 0 \\ (-1)^{n-m+1}\varphi^{n+1} & \partial^n_T \end{pmatrix}$$

and

$$\partial^n_C = \begin{pmatrix} -\partial^{n+1}_K & 0 \\ \varphi^{n+1} & \partial^n_T \end{pmatrix}.$$

It is easy to check that

$$\psi^{n+1} \circ \partial^n = \partial^n \circ \psi^n.$$

In fact the L.S. $= \begin{pmatrix} (-1)^{n-m} & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \partial^{n+1}_K & 0 \\ (-1)^{n-m+1}\varphi^{n+1} & \partial^n_T \end{pmatrix}$

$$= \begin{pmatrix} (-1)^{n-m}\partial^{n+1}_K & 0 \\ (-1)^{n-m+1}\varphi^{n+1} & \partial^n_T \end{pmatrix}$$

And the RS $= \begin{pmatrix} -\partial^{n+1}_K & 0 \\ \varphi^{n+1} & \partial^n_T \end{pmatrix} \begin{pmatrix} (-1)^{n-m+1} & 0 \\ 0 & 1 \end{pmatrix}$

$$= \begin{pmatrix} (-1)^{n-m}\partial^{n+1}_K & 0 \\ (-1)^{n-m+1}\varphi^{n+1} & \partial^n_T \end{pmatrix}.$$

Hence we are done. //