Homework 7, MATH 300 2021-22 Term 2

The numbered questions are from the textbook:

E.B. Saff, A.D. Snider, *Fundamentals of Complex Analysis with Applications to Engineering, Science and Mathematics*, third edition.

We will randomly choose 3-4 questions to mark each time.

1. 4.2.14 (a). You may use the formula $|\int_\Gamma f(z)dz| \leq M\ell(\Gamma)$, where Γ is a contour, f a continuous function on Γ, M a real number such that $|f(z)| \leq M$ for every z on Γ, and $\ell(\Gamma)$ is the length of the contour Γ. This formula will be proved later in the course. You may also use the formula $\ell(\Gamma) = \int_a^b |z'(t)|dt$ where $z(t)$, $a \leq t \leq b$, is a parameterization of Γ.

2. 4.3.2; 4.3.4; 4.3.6; 4.3.12

3. Let Γ be any contour from $z = -3$ to $z = 3$ which never crosses the *negative imaginary axis* and $z = 0$ does not lie on Γ.

   (a) Compute $\int_\Gamma z^{1/2}dz$ where the integrand is the branch

   $$z^{1/2} = \sqrt{r}e^{i\theta/2}, \qquad r > 0, \quad -\pi/2 < \theta < 3\pi/2.$$

   (b) Same as part (a), except now the branch of the integrand is

   $$z^{1/2} = \sqrt{r}e^{i\theta/2}, \qquad r > 0, \quad 3\pi/2 < \theta < 7\pi/2.$$