Homework 4, MATH 300 2021-22 Term 2

The numbered questions are from the textbook:

E.B. Saff, A.D. Snider, *Fundamentals of Complex Analysis with Applications to Engineering, Science and Mathematics*, third edition.

We will randomly choose 3-4 questions to mark each time.

1. 2.4.2; 2.4.12
2. 2.5.6; 2.5.8; 2.5.14; 2.5.18; 2.5.22
3. The following question shows that continuity of the partial derivatives are necessary for Cauchy-Riemann equations to imply differentiability.

   Consider
   $$f(z) = \begin{cases} \dfrac{z^5}{|z|^4}, & z \neq 0 \\ 0, & z = 0. \end{cases}$$

   (a) Represent f in the form of $u(x, y) + iv(x, y)$. (Hint: use binomial theorem.)

   (b) Show that f satisfies the Cauchy-Riemann equations at the origin. (Hint: use the definition of partial derivatives; do not use the quotient rule.)

   (c) Show that f is not differentiable at 0. (Hint: consider the limit from the x-axis and the limit from the ray $\text{Arg}z = \pi/4$.)

   (d) Are $\frac{\partial u}{\partial x}$, $\frac{\partial u}{\partial y}$, $\frac{\partial v}{\partial x}$, $\frac{\partial v}{\partial y}$ continuous at the origin? (To save your time, you only need to check one of the four partial derivatives.)