FAMILY NAME: ___ **Name:**
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## QUIZ 1

Recall the Cauchy estimates

$$|f^{(n)}(a)| \leq n! \frac{M}{R^n} \quad (n = 1, 2, 3, \ldots)$$

for an analytic function on the closed disc $\overline{B}(a,\, R)$ where $M$ is an upper bound for $|f(\zeta)|$ on the circle $C_R$ of radius $R$ around $a$.

A function is said to be *entire* if it is defined and analytic on the entire complex plane.

(1) Show using the Cauchy estimates that a bounded entire function is necessarily constant. (This is called *Louisville's theorem*)

(2) Let $a$ be a point of a region $\Omega$ and $f$ an analytic function on $\Omega \setminus \{a\}$ such that $(z-a)f(z) \to 0$ as $z \to a$. Show that $f$ can be extended to an analytic function on $\Omega$. (This is called *Riemann's removable singularities theorem*). [Hint: Use a homework problem you did involving an integral formula.]