HW 8

Due on March 15, 2017 (in class).

Laurent Expansions.

(1) Let $0 < |a| < |b|$, and consider

$$f(z) = \frac{1}{(z-a)(z-b)}.$$

(a) Expand $f(z)$ as a Laurent series in $0 < |a| < |z| < |b|$.

(b) Expand $f(z)$ as Laurent series for $|z| > |b|$.

(2) Let $\sum_{-\infty}^{\infty} a_n z^n$ and $\sum_{-\infty}^{\infty} b_n z^n$ be two Laurent expansions which converge in the same annulus $A(r, s)$ (centered at 0) $0 < r < s$. What is the Laurent expansion of the product of the functions represented by them?

(3) What is the Laurent expansion of the function

$$f(z) = e^{c(c+\frac{1}{z})}$$

which is regular on $\mathbf{C}^* = \mathbf{C} \setminus \{0\}$?

Residues.

(4) Determine the residues of

(a) $\dfrac{1}{\sin z}$ at $z = k\pi$, $k \in \mathbb{Z}$,

(b) $\dfrac{z}{(z-1)(z-2)^2}$ at $z = 1$ and $z = 2$,

(c) $\tan z$ at $z = \dfrac{\pi}{2} + k\pi$, $k \in \mathbb{Z}$.

(5) Let $R(x) = \dfrac{a_0 + a_1 x + \cdots + a_m x^m}{b_0 + b_1 x + \cdots + b_k x^k}$ be a rational function with real coefficients, whose denominator vanishes for no real value of x, the degree of the denominator exceeding that of the numerator by at least two ($a_m \neq 0$, $b_k \neq 0$, $k \geq m + 2$). Show that the real integral

$$\int_{-\infty}^{\infty} R(x)dx$$

converges and is equal to $2\pi i$ times the sum S of the residues of $R(z)$ at its poles in the upper half-plane.