# HW 2

Due on Jan 18, 2017 (in class).

**Series.** For a power series $\sum_n c_n(z-a)^n$ with radius of convergence $r$, the *circle of convergence* is the circle $\{z \mid |z-a| = r\}$.

(1) If $r > 0$ is the radius of convergence of the series $\sum_{n=0}^{\infty} a_n z^n$, and if at a point $z_0$, $|z_0| = r$, the series converges absolutely, show that $\sum_{n=0}^{\infty} a_n z^n$ converges absolutely and uniformly for $|z| \leq r$

(2) At which point on the circle of convergence does the power series

$$\sum_{n=1}^{\infty} \frac{z^n}{n}$$

converge?

(3) Discuss the uniform convergence of the series

$$\sum \frac{x}{n(1+nx^2)}$$

for real values of $x$.

(4) If $U = u_1 + u_2 + \cdots +$, $V = v_1 + v_2 + \ldots$ are convergent series, prove that $UV = u_1v_1 + (u_1v_2 + u_2v_1) + (u_1v_3 + u_2v_2 + u_3v_1) + \ldots$ provided that at least one of the series is absolutely convergent. (Hint: Modify the well-known argument when both the series are absolutely convergent.)

**Geometry and function theory.**

(5) Let $a \neq 0$ be a complex number. Show that all circles that pass through $a$ and $1/\bar{a}$ intersect the circle $|z| = 1$ at right angles.

(6) If $|a| < 1$, show that $f(z) = \frac{z-a}{1-\bar{a}z}$ is a one-to-one map of the unit disc onto itself. Compute the inverse of $f$ and show that $|f(z)| = 1$ if $|z| = 1$.