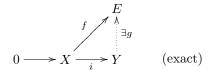
Name: \_\_\_\_\_

## QUIZ 2

Throughout  $\mathscr{A}$  is an abelian category.

**Definitions** Let  $E \in \mathscr{A}$ . *E* is said to be an *an injective object* of  $\mathscr{A}$  (or simply an *injective*) if given a monomorphism  $i: X \hookrightarrow Y$  in  $\mathscr{A}$  and a map  $f: X \to E$ , there exists a map  $g: Y \to E$  such that  $f = g \circ i$ .



The category  $\mathscr{A}$  is said to have *enough injectives* if given an object X in  $\mathscr{A}$  there exists an injective object E in  $\mathscr{A}$  and a monomorphism  $X \hookrightarrow E$  in  $\mathscr{A}$ .

- (1) The dual notion to injective is *projective*. The dual notion to enough injectives is *enough projectives*.
  - (a) Give a direct definition of a *projective object* in  $\mathscr{A}$ .

(b) Give a direct definition of an abelian category with enough projectives.

- (2) Let E be an injective object in  $\mathscr{A}$ .
  - (a) Show that if  $i: E \hookrightarrow X$  is a monomorphism, then E is a direct summand of X. In other words show that  $X = E \oplus E'$  for some object E' in  $\mathscr{A}$ . This means you have to show that there exists an object  $E' \in \mathscr{A}$  and a monomorphism  $j: E' \hookrightarrow X$  such that given maps  $\alpha: E \to Y$  and  $\beta: E' \to Y$ , there exists a unique map  $f: X \to Y$  such that  $f \circ i = \alpha$  and  $f \circ j = \beta$ .

(b) Formulate the dual of the above statement. (Do not give the definition of a direct summand or a direct sum again. It has already been given.)