Name: ________________________________

QUIZ 2

Throughout $\mathscr{A}$ is an abelian category.

Definitions Let $E \in \mathscr{A}$. E is said to be an *an injective object* of $\mathscr{A}$ (or simply an *injective*) if given a monomorphism $i\colon X \hookrightarrow Y$ in $\mathscr{A}$ and a map $f\colon X \to E$, there exists a map $g\colon Y \to E$ such that $f = g \circ i$.

$$\begin{array}{ccccc} & & & & E \\ & & & \nearrow^{f} & \uparrow{\scriptstyle \exists g} \\ 0 & \longrightarrow & X & \xrightarrow[i]{} & Y \end{array} \qquad \text{(exact)}$$

The category $\mathscr{A}$ is said to have *enough injectives* if given an object X in $\mathscr{A}$ there exists an injective object E in $\mathscr{A}$ and a monomorphism $X \hookrightarrow E$ in $\mathscr{A}$.

(1) The dual notion to injective is *projective*. The dual notion to enough injectives is *enough projectives*.

(a) Give a direct definition of a *projective object* in $\mathscr{A}$.

(b) Give a direct definition of an abelian category with enough projectives.

(2) Let E be an injective object in $\mathscr{A}$.

(a) Show that if $i\colon E \hookrightarrow X$ is a monomorphism, then E is a direct summand of X. In other words show that $X = E \oplus E'$ for some object E' in $\mathscr{A}$. This means you have to show that there exists an object $E' \in \mathscr{A}$ and a monomorphism $j\colon E' \hookrightarrow X$ such that given maps $\alpha\colon E \to Y$ and $\beta\colon E' \to Y$, there exists a unique map $f\colon X \to Y$ such that $f \circ i = \alpha$ and $f \circ j = \beta$.

(b) Formulate the dual of the above statement. (Do not give the definition of a direct summand or a direct sum again. It has already been given.)