QUIZ 4

Date: March 29, 2021

1) Let I be an open interval in $\mathbf{R}$, $(\tau, \boldsymbol{a})$ a point in $I \times \mathbf{R}^n$, and $A\colon I \to M_n(\mathbf{R})$ a continuous function. Show using the *fundamental estimate* (see Problem 8, HW 4) that I is an interval of existence for the solution of the initial value problem

$$\dot{\boldsymbol{x}} = A(t)\boldsymbol{x}, \qquad \boldsymbol{x}(\tau) = \boldsymbol{a}.$$

Hint: Pick a compact interval J such that $\tau \in J \subset I$. Show that $\|A(t)\|_\circ$ is bounded on J. Show that $\boldsymbol{\psi}\colon J \to \mathbf{R}^n$, where $\boldsymbol{\psi}(t) \equiv \boldsymbol{a}$, is an ε-approximate solution of our DE for a suitable ε and find suitable compact sets in $I \times \mathbf{R}^n$ from which the solution is forced to exit from the lateral ends.

2) Can the vector field $v\colon \mathbf{R} \to \mathbf{R}$ given by $v(x) = x^3 - x$ be extended to $\mathbf{S}^1$? Why or why not? (See Problems 4, 5, and 6 of the mid-term). Here $\mathbf{S}^1$ is the unit circle in $\mathbf{R}^2$ centred at the origin with its standard structure as a differential manifold.