QUIZ 3
MARCH 15, 2021

1) Let A, X and Y be metric spaces. Suppose $\{f_\alpha\}_{\alpha\in A}$ is a family of continuous maps $f_\alpha\colon X \to Y$ such that

$$\lim_{\alpha\to\alpha_0} \sup_{x\in X} d_Y(f_\alpha(x), f_{\alpha_0}(x)) = 0$$

for every $\alpha_0 \in A$. Show that the map $F\colon A \times X \to Y$ given by

$$F(\alpha, x) = f_\alpha(x) \qquad ((\alpha, x) \in A \times X)$$

is continuous, where the metric on $A \times X$ is

$$d_{A\times X} = ((\alpha, x), (\alpha', x')) = d_A(\alpha, \alpha') + d_X(x, x').$$

2) Let

$$x^{(n)} = F(x, x', x'', \ldots, x^{(n-1)})$$

be an n^{th} order autonomous (scalar) differential equation, where F is $\mathscr{C}^\infty$ on $\mathbf{R}^n$. Show that if $\sin(mt)$ is a solution to the equation for some non-zero integer m then so is $\cos(mt)$.