

QUIZ 5 (ANALYSIS II)

Mar 13, 2020 (in tutorial)

Name: Solutions

Let $\mathbf{p} = (a, b)$ be a point in \mathbf{R}^2 , and let \mathbf{u}, \mathbf{v} be unit vectors in \mathbf{R}^2 which are not multiples of each other (i.e they are linearly independent). Let h, k be non-zero real numbers and $P = P(\mathbf{u}, \mathbf{v}, h, k)$ be the closed set consisting of points of the form $\mathbf{p} + s\mathbf{u} + t\mathbf{v}$, for s a real number in the closed interval I whose end points are 0 and h , and t a real number in the closed interval J whose end points are 0 and k . Let P° be the interior of P , i.e P° consists of points $\mathbf{p} + s\mathbf{u} + t\mathbf{v}$, with s and t in the interior of the intervals I and J respectively.

Let U be an open neighbourhood of \mathbf{p} and $f: U \rightarrow \mathbf{R}$ a map. Define

$$\Delta(P, f) = f(\mathbf{p} + h\mathbf{u} + k\mathbf{v}) - f(\mathbf{p} + k\mathbf{v}) - f(\mathbf{p} + h\mathbf{u}) + f(\mathbf{p})$$



[Should have said “whenever $P \subset U$ ”.]

1. Suppose $D_{\mathbf{u}}f$ and $D_{\mathbf{v}\mathbf{u}}f$ exist on U and $P = P(\mathbf{u}, \mathbf{v}, h, k)$ lies in U . Show that there exists a point $\mathbf{q} \in P^\circ$ such that

$$\Delta(P, f) = hk(D_{\mathbf{v}\mathbf{u}}f)(\mathbf{q}).$$

Solution: This is *mutatis mutandis* the proof of Lemma 2.1.2 of Lecture 11. \square

2. Suppose $D_{\mathbf{u}}f, D_{\mathbf{v}}f, D_{\mathbf{v}\mathbf{u}}f$ exist on U and $D_{\mathbf{v}\mathbf{u}}f$ is continuous at \mathbf{p} . Show that $(D_{\mathbf{u}\mathbf{v}}f)(\mathbf{p})$ exists and

$$(D_{\mathbf{u}\mathbf{v}}f)(\mathbf{p}) = (D_{\mathbf{v}\mathbf{u}}f)(\mathbf{p}).$$

Solution: This is *mutatis mutandis* the proof of Theorem 2.1.3 of Lecture 11. \square