QUIZ 5 (ANALYSIS II)

Mar 13, 2020 (in tutorial) Name: ______________________________

Let $\boldsymbol{p} = (a, b)$ be a point in $\mathbf{R}^2$, and let $\boldsymbol{u}$, $\boldsymbol{v}$ be unit vectors in $\mathbf{R}^2$ which are not multiples of each other (i.e they are linearly independent). Let h, k be non-zero real numbers and $P = P(\boldsymbol{u}, \boldsymbol{v}, h, k)$ be the closed set consisting of points of the form $\boldsymbol{p} + s\boldsymbol{u} + t\boldsymbol{v}$, for s a real number in the closed interval I whose end points are 0 and h, and t a real number in the closed interval J whose end points are 0 and k. Let P° be the interior of P, i.e P° consists of points $\boldsymbol{p} + s\boldsymbol{u} + t\boldsymbol{v}$, with s and t in the interior of the intervals I and J respectively.

Let U be an open neighbourhood of $\boldsymbol{p}$ and $f\colon U \to \mathbf{R}$ a map. Define

$$\Delta(P, f) = f(\boldsymbol{p} + h\boldsymbol{u} + k\boldsymbol{v}) - f(\boldsymbol{p} + k\boldsymbol{v}) - f(\boldsymbol{p} + h\boldsymbol{u}) + f(\boldsymbol{p})$$

[Should have said "whenever $P \subset U$".]

1. Suppose $D_{\boldsymbol{u}} f$ and $D_{\boldsymbol{v}\boldsymbol{u}} f$ exist on U and $P = P(\boldsymbol{u}, \boldsymbol{v}, h, k)$ lies in U. Show that there exists a point $\boldsymbol{q} \in P^\circ$ such that

$$\Delta(P, f) = hk(D_{\boldsymbol{v}\boldsymbol{u}} f)(\boldsymbol{q}).$$

2. Suppose $D_{\boldsymbol{u}}f$, $D_{\boldsymbol{v}}f$, $D_{\boldsymbol{vu}}f$ exist on U and $D_{\boldsymbol{vu}}f$ is continuous at $\boldsymbol{p}$. Show that $(D_{\boldsymbol{uv}}f)(\boldsymbol{p})$ exists and

$$(D_{\boldsymbol{uv}}f)(\boldsymbol{p}) = (D_{\boldsymbol{vu}}f)(\boldsymbol{p}).$$

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