QUIZ 4 (ANALYSIS II)

Feb 14, 2020 (in tutorial) Name: ______________________________

In what follows, $\mathbf{K} \in \{\mathbf{R}, \mathbf{C}\}$. The symbol δ_{ij} is the Kronecker symbol and feel free to ask the TA's what it means if you don't know. Finally, $\mathbf{R}_+ = [0, \infty)$.

(1) Let $(V, \| \|)$ be a finite dimensional normed linear space over $\mathbf{K}$ and W a vector subspace of V. Show that W is closed in V.

(2) Let $\langle\ ,\ \rangle$ be an inner product on a finite dimensional $\mathbf{K}$-vector space V and let $\|\ \|\colon V \to \mathbf{R}_+$ be the usual norm, namely $\|v\| = \langle v, v\rangle^{\frac{1}{2}}$. Let W be a subspace of V and $w_1, \ldots, w_k$ a basis of W such that $\langle w_i, w_j\rangle = \delta_{ij}$ for $i, j \in \{1, \ldots, k\}$. Let $\pi\colon V \to W$ be the map $\pi(v) = \sum_{i=1}^{k} \langle v, w_i\rangle w_i$. Fix $v \in V$ and define $f_v\colon W \to \mathbf{R}_+$ by the formula $f_v(w) = \|w - v\|$. Show that

(a) f_v is continuous on W;

(b) f_v attains its minimum at $\pi(v)$ and if $f_v(w) = f_v(\pi(v))$ for some $w \in W$, then $w = \pi(v)$.

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