# ANALYSIS II
# QUIZ 1

Jan 17, 2020 (in tutorial)

Name: Solutions

The Quiz has two questions. The second question is on the next page.
Let $\mathbf{K} \in \{\mathbf{R}, \mathbf{C}\}$ and let $(V, \| \ \|)$ be a normed linear space over $\mathbf{K}$.

(1) Show that

$$\Big| \|x\| - \|y\| \Big| \leq \|x - y\| \qquad (x, y \in V).$$

**Solution:** We have $x = (x - y) + y$, whence $\|x\| \leq \|x - y\| + \|y\|$. This gives $\|x\| - \|y\| \leq \|x - y\|$, by symmetry, $\|y\| - \|x\| \leq \|y - x\| = \|x - y\|$. Thus $\pm(\|x\| - \|y\|) \leq \|x - y\|$, whence $|\|x\| - \|y\|| \leq \|x - y\|$. $\square$

(2) Show that $\| \ \| \colon V \to \mathbf{R}$ is continuous.

**Solution:** Let $f(x) = \|x\|$ for $x \in V$, i.e. write $f$ instead of $\| \ \|$. Let $v \in V$. Then, by Problem (1),

$$|f(x) - f(v)| = \Big| \|x\| - \|v\| \Big| \leq \|x - v\|.$$

Given $\epsilon > 0$, choose $\delta = \epsilon$. If $\|x - y\| < \delta$, the above inequality shows that $|f(x) - f(v)| \leq \delta = \epsilon$. This means $f$ is continuous, i.e. $\| \ \|$ is continuous. $\square$