WEEK 8 (OCT 31-NOV 6)

Below is a screenshot of the calendar from the syllabus. It has been modified to take care of the fact that we have fallen behind just a bit. Note that the schedule is tentative.

Calendar

This is a tentative schedule for the term (modified on October 23).

Week	Textbook Section	Evaluation	Note
Sep 12–Sep 18	2		
Sep 19–Sep 25	2		
Sep 26–Oct 2	3	PS 1 due on Oct 2	
Oct 3–Oct 9	4.1, 5		
Oct 10–Oct 16	5	PS 2 due on Oct 16	No lecture on Oct 10
Oct 17–Oct 23	5		
Oct 24–Oct 30	5, 7	Midterm project due on Oct 24	
Oct 31–Nov 6	7, 8	PS 3 due on Nov 6	
Nov 7–Nov 13			Reading week
Nov 14–Nov 20	9	PS 4 due on Nov 20	
Nov 21–Nov 27	9		
Nov 28–Dec 4	10	PS 5 due on Dec 4	
Dec 5–Dec 9	10		Make up class on Thursday

Week 8. We will do examples of the Inclusion-Exclusion formula (see the **Problems worth thinking about** section below). Then we will start a new topic, namely *generating functions* which is Chapter 8 of the textbook.

1. Read the basic definitions in Section 8.1.
2. Read Section 8.2

Problems worth thinking about. Here are some problems that you may wish to work on. They are Inclusion-Exclusion and from also combinatorics (a lattice path approach to a problem in the mid-term).

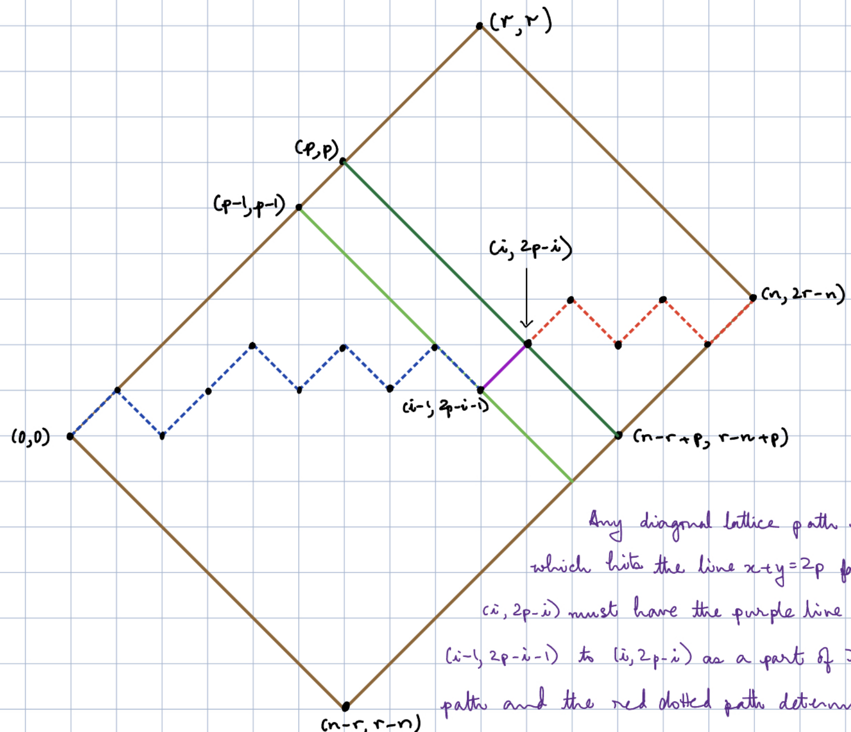
1. Let $1 \leq p \leq r \leq n$ be integers. Think about the following picture involving diagonal lattice paths The claim is that it givse a visual proof of the identity $\binom{n}{r} = \sum_{i=p}^{n-r+p} \binom{i-1}{p-1}\binom{n-r}{r-p}$. Recall that you were asked to prove this identity in the midterm. Can you write out a different proof using the picture?

Visual proof that $\binom{n}{r} = \sum_{i=p}^{n-r+p} \binom{i-1}{p-1}\binom{n-i}{r-p}$

The number of diagonal paths from $(0,0)$ to $(n, 2r-n)$ $= \binom{n}{r}$.

The number of blue paths $= \binom{i-1}{p-1}$ $\Big\}$ for fixed $i \in \{p, p+1, \dots, n-r+p\}$.

The number of red paths $= \binom{n-i}{r-p}$



Any diagonal lattice path from $(0,0)$ to $(n, 2r-n)$ which hits the line $x+y=2p$ for the first time at $(i, 2p-i)$ must have the purple line segment from $(i-1, 2p-i-1)$ to $(i, 2p-i)$ as a part of it. The blue dotted path and the red dotted path determine such paths.

2. Let $m \geq n \geq 1$. How many *surjective* maps are there from $[m]$ to $[n]$? **Hint:** Use the Inclusion-Exclusion formula with A_i, $i \in [n]$ being the number of functions $f\colon [m] \to [n]$ such that $f(j) \neq i$ for any $j \in [m]$. For X use the set of functions of the form $f\colon [m] \to [n]$ and apply I-E to $X \setminus (A_1 \cup \cdots \cup A_n)$

3. Let X be a set with $|X| = n$. A permutation of X is bijective map $f\colon X \to X$ (in other words f is injective and surjective). A *derangement* of X is a permutation such that $f(x) \neq x$ for any $x \in X$. How many derangements of X are there? **Hint:** For simplicity, take $X = [n]$. For $i \in [n]$ let A_i be the set of permutations $f\colon X \to X$ such that $f(i) = i$. Apply Inclusion-Exclusion to $P \setminus (A_1 \cup \cdots \cup A_n)$, where P is the set of permutations on X.