WEEK 6 (OCT 17-OCT 23)

The midterm will be posted this week! Make sure you start on it early, since extensions will not be given. It should not take longer than the usual 90 minute midterm. But you will have many days to do it. With access to notes and books.

Below is a screenshot of the calendar from the syllabus. I am throwing this in case you have missed the calendar in the syllabus. Note that the schedule is tentative. We will continue with Graph Theory (Chapter 5) this week.

Calendar

This is a tentative schedule for the term.

Week	Textbook Section	Evaluation	Note
Sep 12–Sep 18	2		
Sep 19–Sep 25	2		
Sep 26–Oct 2	3	PS 1 due on Oct 2	
Oct 3–Oct 9	4.1, 5		
Oct 10–Oct 16	5	PS 2 due on Oct 16	No lecture on Oct 10
Oct 17–Oct 23	5		
Oct 24–Oct 30	7	Midterm project due on Oct 24	
Oct 31–Nov 6	8	PS 3 due on Nov 6	
Nov 7–Nov 13			Reading week
Nov 14–Nov 20	9	PS 4 due on Nov 20	
Nov 21–Nov 27	9		
Nov 28–Dec 4	10	PS 5 due on Dec 4	
Dec 5–Dec 9	10		Make up class on Thursday

Week 6. The plan is to talk do Eulerian and Hamiltonian graphs and colouring theorems. Things you need to do:

1. **Start on your midterm early!** I should have it on Crowdmark at the latest by Thursday, but it could be as early as Tuesday. Given the amount of time being given for the midterm, I will not be accepting requests for extension of the date.
2. Read Chapter 5. In particular
   - read the proof that a graph is eulerian if and only if it is connected and the degree of every vertex is even.
   - Read the definition of a hamiltonian graph and see the statement of **Dirac's theorem (Theorem 5.18 of the text)**.
3. Look up the definition of Hamilton and Euler paths. Look up also the statement of Euler's result, namely that a graph has a cycle which traverses each edge exactly once if and only if it is connected and every vertex has even degree. This of course means looking up the definitions of all the new terms thrown in (cycle, edge, vertex, degree, connected)

Problems worth thinking about. Here are some problems you might wish to think about. You are not expected to submit the solutions of these problems. They are for practice, and for helping you understand the lectures. Even if you do not succeed in solving them, puzzling over them will be very helpful. You can ask for help with these during the various office hours.

1. Look up the definition of a **planar graph**. A **complete graph** with n vertices, is a graph (with n vertices) such that there is an edge between any two distinct vertices. They are all isomorphic to

$$K_n := (\{1, 2, \ldots, n\}, \{\{i, j\} \mid 1 \le i < j \le n\}).$$

Show that K_5 is not planar [**Hint:** See **Theorem 5.33** of the text.]

2. Show that a planar graph has a vertex of degree at most 5

3. Let $n \ge m \ge 0$. Give a combinatorial proof that

$$2^{n-m}\binom{n}{m} = \sum_{r=m}^{n} \binom{n}{r}\binom{r}{m}.$$

Hint: Form a committee from n members, with the requirement that there is subcommittee of the committee with m members.