WEEK 4 (OCT 3-OCT 9)

Below is a screenshot of the calendar from the syllabus. I am throwing this in case you have missed the calendar in the syllabus. Note that the schedule is tentative. So far we have been keeping to schedule, and I plan to do so the coming week too.

Calendar

This is a tentative schedule for the term.

Week	Textbook Section	Evaluation	Note
Sep 12–Sep 18	2		
Sep 19–Sep 25	2		
Sep 26–Oct 2	3	PS 1 due on Oct 2	
Oct 3–Oct 9	4.1, 5		
Oct 10–Oct 16	5	PS 2 due on Oct 16	No lecture on Oct 10
Oct 17–Oct 23	5		
Oct 24–Oct 30	7	Midterm project due on Oct 24	
Oct 31–Nov 6	8	PS 3 due on Nov 6	
Nov 7–Nov 13			Reading week
Nov 14–Nov 20	9	PS 4 due on Nov 20	
Nov 21–Nov 27	9		
Nov 28–Dec 4	10	PS 5 due on Dec 4	
Dec 5–Dec 9	10		Make up class on Thursday

Week 4. The plan is cover The Pigeon Hole Principle and then spend some time (approximately three lectures) on Graph Theory (Chapter 5). When covering the Pigeon Hole Principle, we will also state and prove the *Strong Form* of this principle, namely if one has has to put d objects into n boxes and $d > n(m-1)$, then at least one box has m or more objects.

This is what you need to know for the next week.

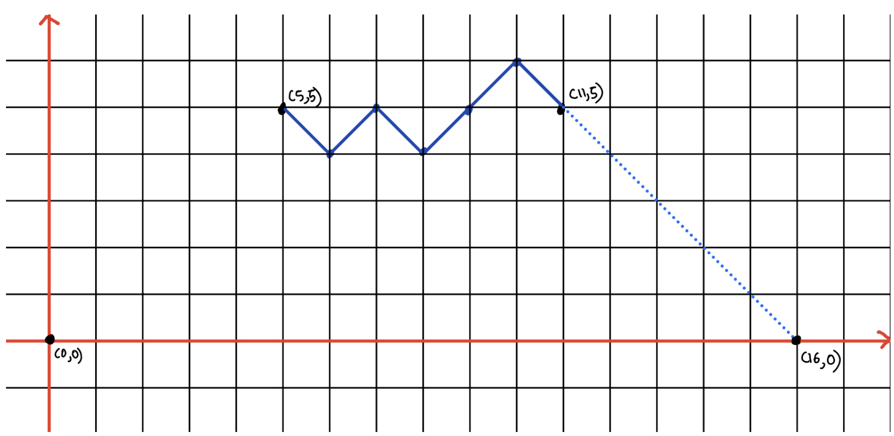
1. You should be comfortable with recursion, induction, and basic counting principles.

2. Look up the The Pigeon Hole Principle and the Erdoös-Szekeres Theorem.

3. Start reading Chapter 5. Look up the definition of Hamilton and Euler paths. Look up also the statement of Euler's result, namely that a graph has a cycle which traverses each edge exactly once if and only if it is connected and every vertex has even degree. This of course means looking up the definitions of all the new terms thrown in (cycle, edge, vertex, degree, connected)

Problems worth thinking about. Here are some problems you might wish to think about. I have not included graph theory problems in this set. I will next week. You are not expected to submit the solutions of these problems. They are for practice, and for helping you understand the lectures. Even if you do not succeed in solving them, puzzling over them will be very helpful. You can ask for help with these during the various office hours.

1. **The Tower of Hanoi Problem.** Three pegs are stuck on a board. On one of these pegs is a pile of disks graduated in size, the smallest being on top. The object of this puzzle is to transfer the pile to one of the other two pegs by moving the disks one at a time from one peg to another in such a way that a disk is never placed on top of a smaller disk.
   (a) Let s_n be the minimal number of moves needed to transfer a pile of n disks. Show that s_n satisfies the recursive relation $s_{n+1} = 2s_n + 1$, $n \in \mathbf{N}$, $s_1 = 1$.
   (b) Show, using induction, that $s_n = 2^n - 1$.
2. Suppose you have a grid consisting of 4 equidistant horizontal lines and 7 equidistant vertical lines and we use three colours to colour the 28 points of intersection between these lines. Call these points at the intersections *grid points*
   (a) Show that in every colouring scheme, each vertical line has two or more grid points of the same colour.
   (b) Suppose every vertical line has at least two grid points which are coloured green. Show that there are two vertical lines and two horizontal lines such that the four grid points these four lines define (via intersections) are all coloured green.
3. How many diagonal lattice paths are there from (5, 5) to (16, 0) which pass through the point (11, 5)?



4. Consider the set $[n] = \{1, 2, \dots, n\}$.
   (a) Show that the number of functions $f\colon [n] \to S$, with $|S| = d$ is d^n.
   (b) Show that there are 6^n ways of finding 5 subsets A_1, A_2, A_3, A_4, and A_5 of $[n]$ such that $A_1 \subset A_2 \subset A_3 \subset A_4 \subset A_5$.
   (c) How many of the ways in (b) are such that $A_3 \neq A_4$?