

WEEK 12 (DEC 5-DEC 11)

Review during make up lecture. The university has designated Thursday, December 8, as the make up for Thanksgiving Monday. The schedule is the usual Monday schedule. We will use that lecture time for review for the final exam. I will answer (as well as I can), questions you may have from material in the course. You can ask for solutions of questions in the Problem Sets, the plans, or problems you find in various books on the topics taught.

Final Exam Schedule. The final is on December 15 from 7:00 PM to 10:00 PM. The exam will be held in four rooms. Please check the exam schedule [here](#) for the room you have to write your exam in.

Week 12. We will do independence (of events and of random variables), Bernoulli trials, the Binomial distribution, and the variance of a random variable.

Tasks. Read about the topics listed above. Start collecting problems you may wish to ask on Thursday.

Problems worth thinking about. Here are some problems that you may wish to work on.

1. Suppose that we have found that the word “Ferrari” occurs in 250 of 2500 messages known to be spam and in 5 of 1000 messages known not to be spam. Assume that these proportions are a good estimate for certain obvious conditional probabilities. Estimate the probability that an incoming message containing the word “Ferrari” is spam. You may assume that a message coming in has an equal chance of being spam or not spam.
2. You have three different kinds of candies, say red, blue, and green, and 10 of each kind. What is the probability that of you randomly handing out 5 blue, 2 green, and 3 red candies to a friend?
3. A string is chosen at random from the set of all binary strings of length n . Show that the expected number of zeroes in the string is $\frac{n}{2}$.
4. A group of nine people sit randomly around a round table. In the group there is a brother and sister pair. What is the probability that the brother and sister sit next to each other? (Remember our conventions from the early lectures about round table seating arrangements.)

Connected components of a graph. Recall that in Lecture 9 we defined the notion of a connected component of a graph $G = (V, E)$. It is a maximal connected subgraph of G . It is clear that if x and y are two distinct vertices of V then x and y lie in distinct connected components of G if and only if there is no path connecting x and y .

5. Let $G = (V, E)$ be a graph and suppose $|V| = 26$. Show that if G has three connected components, then there is at least one component which has fewer than 9 vertices. Conclude that there is at least one vertex v such that $\deg_G(v) \leq 7$.