WEEK 10 (NOV 21-NOV 27)

Below is a screenshot of the calendar from the syllabus. It has been modified to take care of the fact that we have fallen behind just a bit. Note that the schedule is tentative.

Calendar

This is a tentative schedule for the term (modified on October 23).

Week	Textbook Section	Evaluation	Note
Sep 12–Sep 18	2		
Sep 19–Sep 25	2		
Sep 26–Oct 2	3	PS 1 due on Oct 2	
Oct 3–Oct 9	4.1, 5		
Oct 10–Oct 16	5	PS 2 due on Oct 16	No lecture on Oct 10
Oct 17–Oct 23	5		
Oct 24–Oct 30	5, 7	Midterm project due on Oct 24	
Oct 31–Nov 6	7, 8	PS 3 due on Nov 6	
Nov 7–Nov 13			Reading week
Nov 14–Nov 20	9	PS 4 due on Nov 20	
Nov 21–Nov 27	9		
Nov 28–Dec 4	10	PS 5 due on Dec 4	
Dec 5–Dec 9	10		Make up class on Thursday

Week 10. We will continue with Chapter 9. So far we have done linear homogeneous recurrence relations. We will now work on non-homogeneous linear recurrence relations and also some non-linear recurrence relations (by using generating functions). In greater detail, we will do

1. § 9.4.2
2. § 9.6

Tasks. To make the week easier on yourself, do the following:

- Read Examples 9.14, 9.15. 9.16, and 9.17 in § 9.4.2.
- Read Example 9.24 from §9.6.

Problems worth thinking about. Here are some problems that you may wish to work on.

1. Show (using Newton's Binomial Theorem) that

$$\sqrt{1-4x} = 1 - 2\sum_{n=0}^{\infty} \frac{1}{n+1}\binom{2n}{n} x^{n+1}.$$

2. Show using exponential generating functions that the number of surjective maps from $[n]$ to $[m]$ is $\sum_{k=0}^{n}(-1)^k\binom{m}{k}(m-k)^n$. We proved this in Lecture 14. Conclude that if $n \leq m$ then $\sum_{k=0}^{n}(-1)^k\binom{m}{k}(m-k)^n = 0$. Check this is true for small values of n and m, for example $n = 3$ and $m = 5$.

3. Suppose $(a_n)_{n=0}^{\infty}$ satisfies the recurrence relation $a_n = na_{n-1} + (-1)^n$, $n \in \mathbf{N}$, with $a_0 = 1$. Let $E(x)$ be the exponential generating function of $(a_n)_{n=0}^{\infty}$. Show that $E(x) = e^{-x}/(1-x)$.[1] Show that $a_n = n! \sum_{k=0}^{n} (-1)^k / k!$, $n \in \mathbf{N}_0$. Is the formula familiar from something done earlier in the course?

[1]**Hint:** Use $\sum_{n=1}^{\infty} \frac{1}{n!} a_n x^n = \sum_{n=1}^{\infty} \frac{1}{n!} n a_{n-1} x^n + \sum_{n=1}^{\infty} \frac{1}{n!} (-1)^n x^n$.