MAT344 FALL 2022
PROBLEM SET 1

Due date: Oct 2, 2022 (on Crowdmark by midnight)

In what follows **N**, **Z**, **R** denote the set of positive integers, the set of integers, and the set of real numbers respectively. $\mathbf{N}_0$ denotes the set of non-negative integers.

1. How many arrangements are there of the 7 letters in ADAMANT?

2. Let $k \geq n$. Give a **combinatorial proof** of the identity

$$\sum_{r=0}^{n} \binom{n}{r}\binom{k}{r} = \binom{n+k}{n}.$$

3. Let $n \geq 9$ be an integer. Give a combinatorial proof of the identity

$$\binom{n}{9} = \sum_{k=4}^{n-5} \binom{k-1}{3}\binom{n-k}{5}.$$

4. In how many ways can you distribute 12 identical objects to 5 people if the first person can only have 4 or 5 objects and the second person cannot have more than 3 objects?

Diagonal Lattice Paths. Recall that there is another kind of lattice path, the so-called *diagonal lattice path*, that is common in the literatute. These consist of steps from (i, j) to either $(i+1, j+1)$ or $(i+1, j-1)$. Recall that the number of diagonal lattice maths from $(0, 0)$ to (m, n) is $C(m, (m-n)/2)$ where our convention is that for a real number r, $C(m, r) = 0$ if r does not belong to $\{0, 1, \ldots, m\}$.

5. Let $n \in \mathbf{N}$. Assume that there is a diagonal lattice path from (0, 0) to $(n, 3)$. Prove combinatorially that the number of diagonal lattice paths from $(0, 0)$ to $(n, 3)$ which dip below the line $y = -1$ is $C(n, (n+7)/2)$.

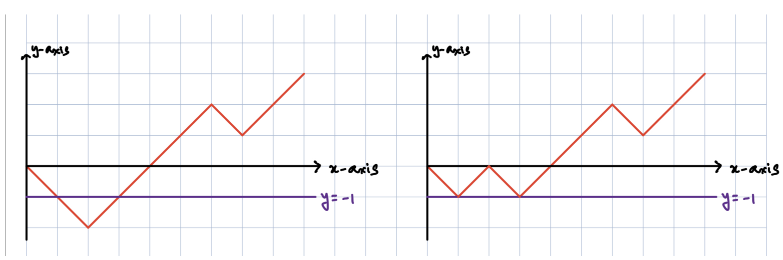

FIGURE 1. The path on the left dips below the line $y = -1$ but the path on the right does not.