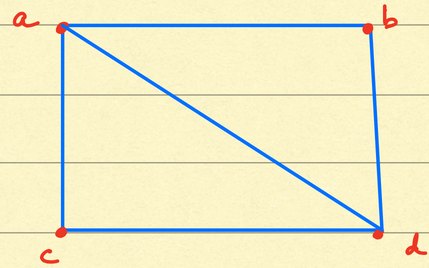
Informal notion of a graph

Through pictures. Have vertices & edges.

1.



Vertices: a, b, c, d

Edges: (a,b) (same as (b,a))

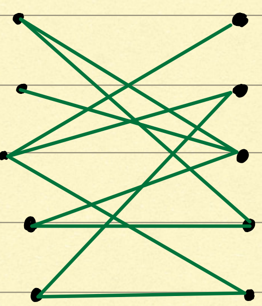
(a,c) (same as (c,a))

(a,d) (same as (d,a))

(b,d) (same as (d,b))

(c,d) (same as (d,c))

2.



For this graph:

Vertices: A, B, C, D, E, a, b, c, d, e.

Edges: There are 10 of them.

(A,c), (A,d), (B,c), (C,a), (C,b), (C,d)

(D,c), (D,d), (E,b), (E,e).

Formal notions

Definition: A graph $G = (V, E)$ is a finite set V along with a set E of unordered pairs of distinct elements of V.

Two edges cannot join same two vertices

V: The "vertices" of G

E: The "edges" of G

If $(a,b) \in E$, i.e., if (a,b) is an edge of G, we say e is <u>incident</u> to a and to b.

In 1., $G=(V,E)$ with

$$V=\{a,b,c,d\},\ E=\{(a,b),(a,c),(a,d),(b,d),(c,d)\}$$

In 2., $G=(V,E)$ with

$$V=\{A,B,C,D,E,a,b,c,d,e\}$$

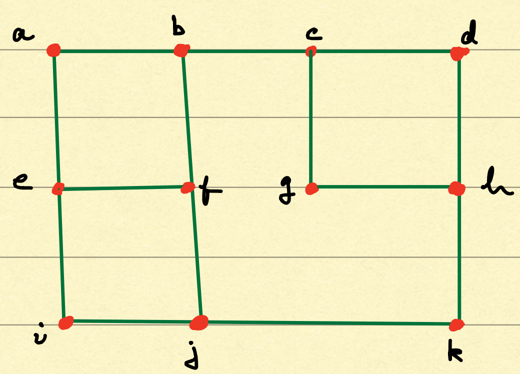
$$E=\left\{\begin{matrix}(A,c), & (A,d), & (B,c), & (C,a), & (C,b), & (C,e) \\ (D,c), & (D,d), & (E,b), & (E,e) & & \end{matrix}\right\}$$

3. Suppose $G=(V,E)$ with

$V=\{a,b,c,d,e,f,g,h,i,j,k\}$ and

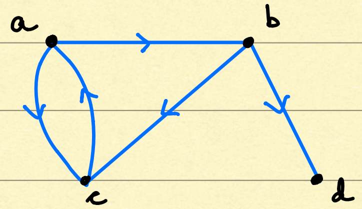
$$E=\left\{\begin{matrix}(a,b), (a,e), (b,c), (b,f), (c,d), (c,g), (d,h) \\ (e,f), (e,i), (f,j), (g,h), (h,k) \\ (i,j), (j,k)\end{matrix}\right\}$$

The pictorial representation of G is



Other pictorial representations are also possible.

Directed graphs:



Sometimes edges are <u>ordered pairs</u> of vertices called <u>directed edges</u>. We can allow two directed edges to join the same two vertices provided they are in <u>different directions</u>!

A directed edge "from" a "to" b is denoted $(\vec{a}, b)$. ordered pair.

A directed graph $G = (V, E)$ consists of a finite set of vertices V and a collection of <u>ordered pairs</u> of vertices E.

In the picture above $G = (V, E)$ with

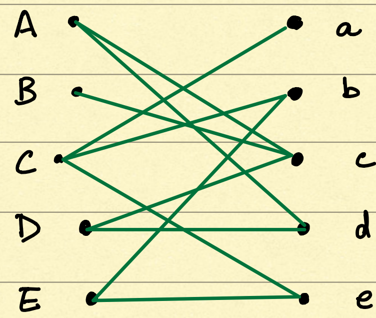
$$V = \{a, b, c, d\}$$

$$E = \{ (\vec{a}, b), (\vec{a}, c), (\vec{b}, c), (\vec{b}, d), (\vec{c}, a) \}$$

Two directed edges joining same vertices

<u>Matching</u>: The graph in example 2 can be used to model the following situation. Suppose we have five people A, B, C, D, E and five jobs a, b, c, d, e and that various people are qualified for various jobs. We want a way of matching in a one-to-one way people with jobs.

Draw a graph with vertices A, B, C, D, E and a, b, c, d, e and connect an upper case letter (i.e., a person) with a lower case letter (i.e., a job) if the person is qualified for the job. Let us say the graph in Example 2 is what the resulting graph looks like.



Is there a feasible solution to the problem? Can we find a way of assigning a person to a job she or he is qualified for such that everybody has a job and all jobs vacancies are filled (one person per job)?

The answer is No because A, B, D are qualified for only two jobs, c and d, and so at least one of them has to sit out. There are other reasons too. Here is another argument. C is the only one qualified to do a, and so C must be matched to a. So C cannot be matched to either b or e (the other jobs C is qualified for). This leaves us with E being the only remaining person qualified to do b and e, and E cannot do both since we are not allowing one person to hold two jobs.

The example in 3 can be used to model a number of situations. We will talk about these (network reliability, street survillance, scheduling meetings) in the next class. In the meanwhile read sections 1.1 and 1.2 to prepare for the next class.