MAT 344 INTRODUCTION TO COMBINATORICS
MIDTERM TEST. FEB 27, 2018. NO AIDS ALLOWED

The maximum grade is 100. You do not have to solve all problems to receive this grade.

(1) (20 marks) Draw two non-isomorphic graphs with six vertices and 10 edges. Prove the two graphs you have drawn are not isomorphic.

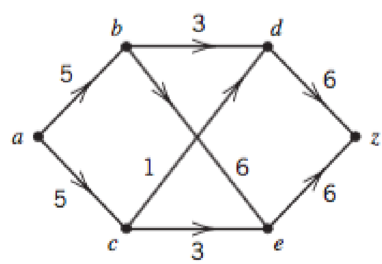
(2) Let K_n denote the complete graph with n vertices, and $K_{m,n}$ the complete bipartite graph with m vertices on the left, and n vertices on the right.
   (a) (10 marks) Show that $K_{2,n}$ is planar for every $n \geq 1$.
   (b) (10 marks) Show that K_5 is not planar by using the corollary to Euler's formula connecting the number of regions, edges, and vertices.

(3) (20 marks) Suppose a graph G has all vertices of degree $\leq d$. Show that its chromatic number $\chi(G)$ is less than or equal to $d + 1$.

(4) (20 marks) Suppose all vertices of a graph have degree 15. Show that the number of edges in G is a multiple of 15.

(5) (20 marks) Let G be a connected planar graph with eight vertices and 12 edges. Show that a planar depiction of G must have at least three regions of degree ≤ 4.

(6) Consider the following directed a-z network N with capacities as displayed.



See next page for the rest of question 6.

(a) (5 marks) Which of the following is not a flow for N? (The picture for N is reproduced below again for your convenience.)

(i) $f(\vec{a},b) = 5,\ f(\vec{a},c) = 5,\ f(\vec{b},d) = 3,\ f(\vec{b},e) = 2,$
$f(\vec{c},d) = 1,\ f(\vec{c},e) = 4,\ f(\vec{d},z) = 4,\ f(\vec{e},z) = 6.$

(ii) $f(\vec{a},b) = 5,\ f(\vec{a},c) = 3,\ f(\vec{b},d) = 3,\ f(\vec{b},e) = 2,$
$f(\vec{c},d) = 1,\ f(\vec{c},e) = 2,\ f(\vec{d},z) = 4,\ f(\vec{e},z) = 4.$

(iii) $f(\vec{a},b) = 5,\ f(\vec{a},c) = 4,\ f(\vec{b},d) = 3,\ f(\vec{b},e) = 2,$
$f(\vec{c},d) = 1,\ f(\vec{c},e) = 3,\ f(\vec{d},z) = 4,\ f(\vec{e},z) = 5.$

(b) (5 marks) For a directed network, define the cut $(P, \overline{P})$ associated with a set P of vertices of the network.

(c) (10 marks) Find a maximum flow for the network N displayed on the previous page (and below) and show it is a maximum flow by finding a minimum a-z cut $(P, \overline{P})$ for that flow. Prove that your cut is indeed the minimum cut for your flow.

So that you do not have to keep flipping this page, here is a picture of N again: