University of Toronto The Faculty of Arts and Science Department of Mathematics

Final Examinations April 2018 MAT344H1S, Introduction to Combinatorics Duration: 3 hrs Maximum grade is 100. No Aids allowed

Last Name (PRINT)	
Given Name(s) (PRINT)	
Student $\#$	

You have to show work to get credit (except in Question 2). The exam has 5 questions, each worth 20 marks. There are 12 pages, including this page. The Questions begin on page 3 and end on page 8. There are four blank pages towards the end and one behind this page for scratch work, and they will not be marked at unless you indicate **clearly** otherwise in the question pages.

Question	Marks
1	/20
2	/20
3 (a)	/10
3 (b)	/10
4 (a)	/10
4(b)	/10
5	/20
TOTAL	/100

1. (20 marks) Verify the identity

$$\sum_{k=0}^{m} \binom{m}{k} \binom{n}{r+k} = \binom{m+n}{m+r}.$$

2. (20 marks) This is a multiple choice question. Unlike all other questions in this exam, you only need to indicate the correct answer, and no solution is required. Suppose $p \ge 2q > 0$. How many ways are there of distributing p identical bags amongst q persons so that everyone has at least two bags?

(A)
$$\binom{p+q-1}{p-1}$$
, (B) $\binom{p-q-1}{p-1}$, (C) $\binom{p-q-1}{q-1}$, (D) $\binom{p+q-1}{q-1}$, (E) $\binom{p}{q}$, (F) $\binom{p+q}{q-1}$, (G) $\binom{p+q}{p-1}$.

3. (a) (10 marks) Find a generating function for the number of integer solutions of 3x + 2y + 9z = r with $x, y, z \ge 0$.

(b) (10 marks) Find the exponential generating function for the number of ways to distribute r distinct pencils amongst five people with each person having an odd number of pencils.

4. (a) (10 marks) Solve the recurrence relation:

$$a_n = 2a_{n-1} + n^2, \quad a_0 = 3.$$

(b) (10 marks) Solve the recurrence relation:

$$a_{n+1} = \sum_{i=0}^{n} a_i a_{n-i}, \quad n \ge 0, \ a_0 = 1.$$

5. (20 marks) How many ways are there of arranging the letters in COMMITTEE so that the two M's are bunched together and O occurs before C?