# University of Toronto <br> The Faculty of Arts and Science <br> Department of Mathematics 

Final Examinations April 2018<br>MAT344H1S, Introduction to Combinatorics<br>Duration: 3 hrs<br>Maximum grade is 100. No Aids allowed

## Last Name (PRINT)

Given Name(s) (PRINT) $\qquad$
Student \#

You have to show work to get credit (except in Question 2). The exam has 5 questions, each worth 20 marks. There are 12 pages, including this page. The Questions begin on page 3 and end on page 8. There are four blank pages towards the end and one behind this page for scratch work, and they will not be marked at unless you indicate clearly otherwise in the question pages.

| Question | Marks |
| :---: | ---: |
| 1 | $/ 20$ |
| 2 | $/ 20$ |
| $3(\mathrm{a})$ | $/ 10$ |
| $3(\mathrm{~b})$ | $/ 10$ |
| $4(\mathrm{a})$ | $/ 10$ |
| $4(\mathrm{~b})$ | $/ 20$ |
| 5 | $/ 100$ |

This page has been left intentionally blank.

1. (20 marks) Verify the identity

$$
\sum_{k=0}^{m}\binom{m}{k}\binom{n}{r+k}=\binom{m+n}{m+r}
$$

2. (20 marks) This is a multiple choice question. Unlike all other questions in this exam, you only need to indicate the correct answer, and no solution is required. Suppose $p \geq 2 q>0$. How many ways are there of distributing $p$ identical bags amongst $q$ persons so that everyone has at least two bags?
(A) $\binom{p+q-1}{p-1}$,
(B) $\binom{p-q-1}{p-1}$,
(C) $\binom{p-q-1}{q-1}$,
(D) $\binom{p+q-1}{q-1}$,
(E) $\binom{p}{q},(\mathrm{~F})\binom{p+q}{q-1}$,
(G) $\binom{p+q}{p-1}$.
3. (a) (10 marks) Find a generating function for the number of integer solutions of $3 x+2 y+$ $9 z=r$ with $x, y, z \geq 0$.
(b) (10 marks) Find the exponential generating function for the number of ways to distribute $r$ distinct pencils amongst five people with each person having an odd number of pencils.
4. (a) (10 marks) Solve the recurrence relation:

$$
a_{n}=2 a_{n-1}+n^{2}, \quad a_{0}=3
$$

(b) (10 marks) Solve the recurrence relation:

$$
a_{n+1}=\sum_{i=0}^{n} a_{i} a_{n-i}, \quad n \geq 0, a_{0}=1
$$

5. (20 marks) How many ways are there of arranging the letters in COMMITTEE so that the two M's are bunched together and O occurs before C?

This page has been left intentionally blank.

This page has been left intentionally blank.

This page has been left intentionally blank.

This page has been left intentionally blank.

