# QUIZ 4
## OCTOBER 10, 2018

Recall from HW-7 that if $X$ is a Banach space, then there is a canonical isometric embedding $J\colon X \hookrightarrow X^{**}$ where $(Jx)(\lambda) = \lambda(x)$. $X$ is said to be *reflexive* if $J$ is an onto map.

(1) Let $X$ be the normed linear space of continuous functions on [0,1] with the norm $\|f\| = \int_0^1 |f(t)|dt$. Show that the linear functional $\Lambda f = f(\frac{1}{2})$ is not bounded.

(2) In this problem we will make the identification $(\ell^1)^* = \ell^\infty$ and identify continuous functionals on $C(X)$, $X$ a compact Hausdorff space, with regular complex Borel measures on $X$. Let $\widehat{\mathbb{N}} = \mathbb{N} \cup \{\infty\}$ be the one point compactification of the discrete topological space $\mathbb{N}$. Consider $\delta_\infty$, the Dirac measure at $\infty$. Show that this is a functional of norm 1 on $c$, the closed subspace of $\ell^\infty$ consisting of convergent sequences. By Hahn Banach this extends to a functional on $\Lambda$ on $\ell^\infty$ with $\|\Lambda\| = 1$. Show that $\Lambda$ does not lie in $J(\ell^1)$, where $J$ is as defined at the beginning of this quiz. Conclude that $\ell^1$ is not reflexive.