Quiz 1

Let X be a set. For a sequence of subsets $\{A_n\}$ of X define

$$\limsup A_n = \bigcap_{n=1}^{\infty} \bigcup_{k=n}^{\infty} A_k$$

and

$$\liminf A_n = \bigcup_{n=1}^{\infty} \bigcap_{k=n}^{\infty} A_k$$

1. Show that

(5 marks) (a) $\limsup A_n = \{x \in X \mid x \in A_n \text{ for infinitely many } n\}$

(5 marks) (b) $\liminf A_n = \left\{ x \in X \,\middle|\, \begin{array}{l} x \in A_n \text{ for all but a finite number} \\ \text{of } n. \end{array} \right\}$

2. (a) If $A_1 \subseteq A_2 \subseteq A_3 \subseteq A_4 \subseteq \dots$ prove that
(5 marks)
$$\limsup A_n = \liminf A_n = \bigcup_{k=1}^{\infty} A_k.$$

(b) If $A_1 \supseteq A_2 \supseteq A_3 \supseteq A_4 \supseteq \dots$ prove that
(5 marks)
$$\limsup A_n = \liminf A_n = \bigcap_{k=1}^{\infty} A_k.$$