

Linear Programming and Combinatorial Optimization

Tutorial 3

February 8, 2019

1. Consider the first LP we wrote in class for the minimum spanning tree problem:

$$\begin{aligned}
 \text{Minimize} \quad & \sum_{e \in E} c_e x_e \\
 \sum_{e \in E} x_e &= |V| - 1 \\
 \sum_{e \text{ crosses } S} x_e &\geq 1 \quad \forall S \subset V \\
 x_e &\geq 0 \quad \forall e \in E
 \end{aligned}$$

Write the dual for this LP and design a primal-dual algorithm. Which combinatorial algorithm does it remind you of?

2. We have seen the maximum matching LP:

$$\begin{aligned}
 \text{Maximize} \quad & \sum_{e \in E} x_e \\
 \sum_{e \sim v} x_e &\geq 1 \quad \forall v \in V \\
 x_e &\geq 0 \quad \forall e \in E
 \end{aligned}$$

Write its dual and give its graph theoretic interpretation.

3. An LP for network flows is given below:

$$\begin{aligned}
 \text{Maximize} \quad & \sum_{u:(s,u) \in E} f_{su} \\
 f_e &\leq c_e \quad \forall e \in E \\
 \sum_{u:(u,v) \in E} f_{uv} - \sum_{w:(v,w) \in E} f_{vw} &= 0 \quad \forall v \in V \\
 f_e &\geq 0
 \end{aligned}$$

Write its dual and give its graph theoretic interpretation.