Assignment 2

March 29, 2017

1. Show that any 3-regular 2-edge-connected graph $G = (V, E)$ (not necessarily bipartite) has a perfect matching. A 2-edge-connected graph has at least two edges across any cut $(S, V \setminus S)$, $S \subset V, S \neq \emptyset$.

2. Could there be several minimizers U in the Tutte-Berge formula? Either give a graph with several sets U achieving the minimum or prove that the set U is unique.

3. **Unique set cover problem:** We are given a set of n elements and a collection of m subsets of the elements. The goal is to pick out a number of subsets so as to maximize the number of *uniquely covered* elements, those that are contained in exactly one of the picked subsets. (Note that the cost or number of subsets picked is not important.)

   (a) Consider the following algorithm: for some number $p < 1$, the algorithm picks each subset independently with probability p. Assuming that every element is contained in exactly F subsets, compute the expected number of uniquely covered elements. For what value of p is this expectation maximized?

   (b) Assuming that each element is contained in at least $F/2$ subsets and at most F subsets, give a constant factor approximation to unique set cover using the algorithm in part 3a.

   (c) Extend the algorithm to obtain an $O(\log m)$ approximation in general, without assumptions on the frequency of any element. (*Hint: Try reducing this case to the case mentioned in part 3b.*)

   (d) Can you improve the approximation from part 3c to a factor of $O(\log n)$? (*Hint: Can you limit the number of sets under consideration to only n?*)

4. Suppose we run the Misra-Gries algorithm on two streams A_1, A_2 and obtain a set of k (element, counter) pairs for each of them. The goal is to merge these k counters to get k counters for a single stream obtained by concatenation of A_1, A_2. Consider the following algorithm for this:

   (a) Combine the two sets of counters, adding up the counters for common elements.

   (b) If more than k counters remain, then let $c =$ value of $(k+1)$th counter based on increasing order of value. Reduce each counter by c and delete all the pairs with non-positive counters.

   Show that the resulting set of (element,counter) pairs satisfy the frequency estimate bound $f_j - \frac{m}{k} \leq \hat{f}_j \leq f_j$ for the concatenated stream.